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MULTIPLE REFLECTION OF SHORT ELECTRIC WAVES FROM SCREENS OF METALLIC RESONATORS.

By W. L. Severinghaus and W. S. Nelms.

THE problem of electrical resonance has been taken up in a great many different forms by many investigators. The oscillator used as a source has usually emitted an uncertain wave-length and been the cause of more or less difficulty. The present work offers a solution to some of these difficulties and a means of getting a short electric wave of any definite length which is much less damped than that of the Righi vibrator. The apparatus is in general similar to that used for selective reflection in the infra-red spectrum, namely, an exciting source, reflecting screens or surfaces, an interferometer and a non-selective receiver.

Apparatus.

The apparatus is most conveniently discussed under three heads, (1) the oscillating system, (2) the reflecting system, (3) the receiving system.

1. *The Oscillating System.*—The vibrator is the ordinary form of Righi vibrator. It consists of two steel spheres set in sockets ground in wood fiber supports which are so arranged that the distance between the spheres can be accurately controlled and made as small as desirable. The supports are devised to allow of quick and easy adjustment to the focus of the mirror for spheres of different diameters, whenever a change in the wave-length is desired. A steady stream of clean paraffin oil flows over the balls and fills the gap between them. Lead wires are brought from the secondary of the induction coil, which is directly under the vibrator, making side sparks in air about a half centimeter. The oil gap is perhaps not more than 0.1 mm., being adjusted to give maximum energy. This vibrator is placed at the focus of a spherical parabolic mirror, of 50 cm. focal length, in order to get sufficient energy, and to

obtain a plane wave front. The vibrator is in general the same as that described in a paper by Woodman and Webb.[1]

The parabolic mirrors used had to be made in the laboratory, since sufficiently large ones could not be bought, and they proved so satisfactory that a brief statement as to their construction may be of interest. Soft plaster of paris was poured into a shallow circular box made by fastening a ring of sheet brass on a flat wooden base. A parabolic templet was rotated about a vertical axis through the center of the box until the plaster had hardened. The parabolic surface thus formed was covered with heavy lead foil making an excellent reflector of the desired focal length. The diameter of the mirror was 83 cm. and the focal length was 50 cm.

One of these mirrors was held in a rack which had freedom of adjustment so that either its height from the floor or the angle it made with the vertical or horizontal could be controlled. The vibrator was placed at

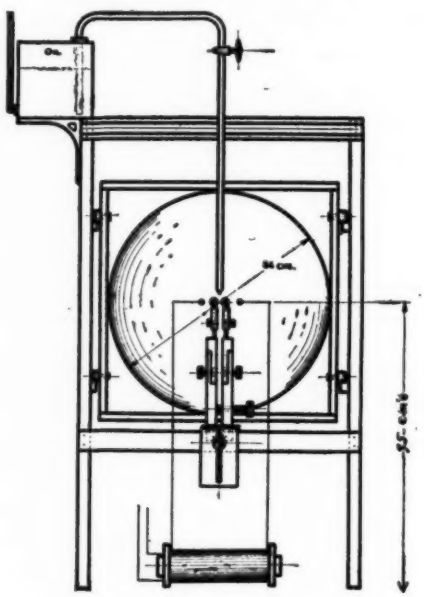


Fig. 1.

Oscillating system.

the focus of the mirror so that the electric vector of the oscillation was in the horizontal plane (see Fig. 1). The vertical as well as the horizontal distribution of energy in the beam on both sides of the principal axis of this mirror is shown in the curves of Fig. 2. The energy falls to half

[1] Woodman and Webb, PHYS. REV., Vol. XXX., p. 563, 1910.

value at about 30 cm. distance from the central axis of the beam, whereas with 12 cm. mirrors it fell to half value at 15 cm. from the axis. This added width of the beam was quite a help in some of the later work with resonator screens.

The mirror is known to affect in some way the character and wave-length of radiation obtained from the oscillator. This is one of the great

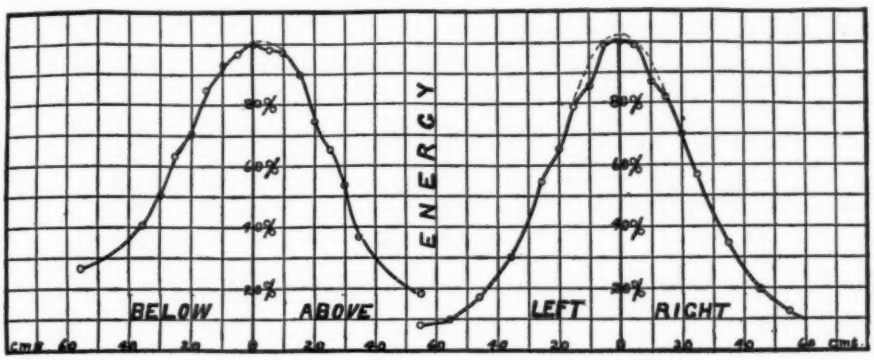


Fig. 2.

Distribution of energy in the beam.

difficulties in using the Righi vibrator as a direct source of energy. It seems to set a limit for the possible wave-length to be otherwise obtained by increasing the size of the spheres. At any rate it makes the increase in energy and in wave-length beyond certain values not at all proportional to the increase in the size of the oscillator. An attempt was made to find the reason for this effect by making some plaster casts on 12 cm. focal length mirrors and covering these forms with lead foil. It was thought that the energy reflected from the mirror back on to the vibrator might cause the trouble. The central portion of the lead foil was therefore removed, so that none of the reflected energy came to the vibrator, but the phenomenon was the same as before. The foil was cut up into strips and the radiation again investigated, but no satisfactory information was obtained as to the cause of the trouble. That the energy and wave-length will increase with the dimensions of the vibrator, when no reflector or a reflector of large focal length is used, was shown by Webb and Woodman, who also performed some experiments on this "mirror action."[1] The effect seems to grow less as the focal length becomes large compared to the wave-length.

It cannot be said that even the large mirror used in this work did not have an effect on the radiation from the oscillator. It did, however, allow

[1] Webb and Woodman, PHYS. REV., Vol. XXIX., p. 100, 1909.

of sufficient variation in the wave-length for our purposes, and that is all we needed as we were not interested in the effect of the dimensions of the oscillating system on the wave-length emitted by it. It is barely possible that the induction coil and lead wires have some effect on the wave-length and damping. For this reason we prefer to speak of the *oscillating system*, with all that may be involved, and merely assume that the wave-length measured by the Boltzmann mirrors is the true wave-length of the emitted energy.

2. *The Reflecting Sytsem.*—The reflecting system consisted first of two rigid wooden stands upon which any desired surfaces could be mounted at S_1 and S_2 (see Fig. 3). After reflection at S_1 and S_2 the energy fell on

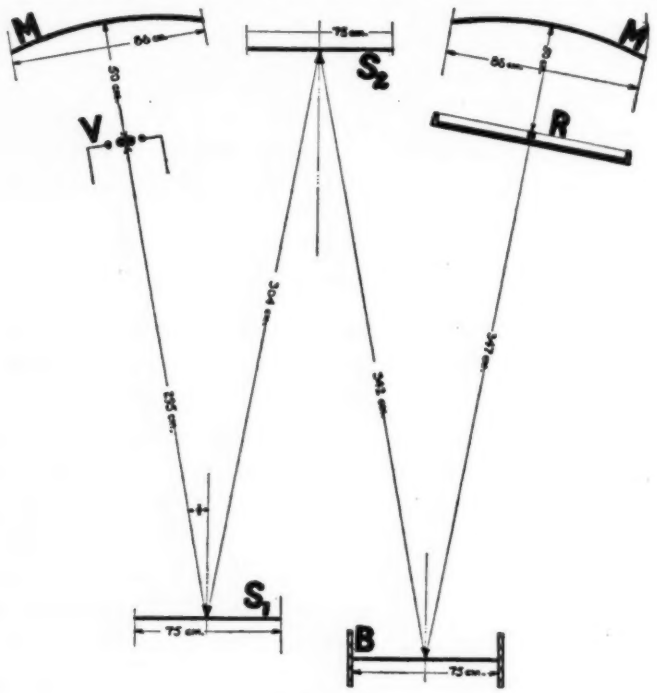


Fig. 3.

the Boltzmann mirrors (B) which consisted of two pieces of plate glass (76 × 38.5 cm.), the upper and the lower half, covered with tin foil. Total reflectors to be used at S_1 or S_2, when desired, were made by covering heavy plate glass with tinfoil and mounting them in wooden frames to fit the stands. The selective resonator screens were in every case made by pasting the tinfoil resonators on tracing cloth. The tracing cloth was thoroughly wetted, stretched over a wooden frame, tacked and dried before the resonators were put on. This gave a very plane surface

which reflected none of the energy and had been found by others to be the same as suspending the resonators in air. These screens as also the total reflectors were 85 × 85 cm., a trifle larger than the area of the vibrator mirror aperture. The angle of incidence of the beam on the screens was 11° and the total length of path from vibrator to receiver was 14.9 meters.

3. *The Receiving System.*—The energy was reflected from the Boltzmann mirrors to a parabolic mirror, exactly similar to the one used with the vibrator, and brought to a focus on a *non-selective* Klemenčič receiver. Observations indicated that this mirror had little or no effect on the wave-length measurements. It seems certain at any rate that the receiver mirror does not exert so large an influence as the vibrator mirror.

The receiver in its final form was made by stretching copper wings (*a*, *a*, Fig. 4) over a wooden frame. These wings were fastened at the

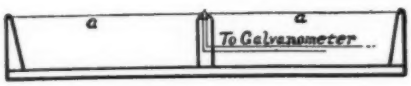


Fig. 4.

Non-selective receiver.

outside ends with sealing wax, while at the center they were sewed on to a base of hard rubber. The thermo-junction consisted of fine platinum and constantan wires about 0.0001 inch in diameter, soldered on to the wings and welded together at the center by a spark from a static machine. The wires leading to the galvanometer were soldered to the under side of the wings at the junction and led out immediately without passing through or along any dielectric. The total width of dielectric in contact with the wings at the center was not more than 4 millimeters, while the total length of the receiver was about 200 centimeters, which was from 12 to 35 times the length of the waves measured. The receiver is for the most part the same as the one perfected by Webb and Woodman[1] and described in their paper.

The reliability of results in wave-length measurements depends very largely on the character of the receiver. It has been shown by a number of investigators[2] that the interference curves obtained with selective receivers are a result of the combination of two frequencies, namely, that of the vibrator and that of the receiver itself. The apparent damping of the oscillation is also greatly affected by the tuned receiver, so that an oscillation which, measured with a non-selective receiver, shows high

[1] Webb and Woodman, l. c., p. 93.

[2] Hull, Blake and Fountain, Ives, Willard and Woodman.

damping, shows very little damping when measured with a tuned receiver. It is then extremely important in the work which is here presented that the receiver be strictly dead beat or non-selective. The receiver described above meets these conditions.

The general plan and dimensions of the apparatus can be seen in Fig. 3. The figure explains itself and throughout the discussion the letters used in the drawing shall be the symbols for the respective parts. Fig. 1 shows the front elevation of the oscillating system. The alignment of the apparatus was made by testing the electrical intensity at various points, and then adjusting the angle of the oscillator and screens until the distribution of energy was symmetrical about the centers of the two screens, Boltzmann mirrors and receiver mirror. With total reflectors at S_1 and S_2 a watch held at the focus of one of the mirrors could be heard very distinctly with the ear at the focus of the other mirror.

METHOD OF WAVE-LENGTH DETERMINATIONS.

All of the work was done with two observers, one reading the galvanometer connected with the main receiver and the other the galvanometer connected with a check receiver. The check receiver was also non-selective and was placed just above the vibrator outside the beam reflected from the oscillator mirror. The use of the check receiver helped to eliminate errors which would otherwise have arisen from a deterioration of the vibrator.

Before proceeding with the wave-length measurement the energy in the beam had to be tested and the vibrator or receiver so adjusted, by slightly raising or lowering, that the energy received from the upper half of the Boltzmann mirrors was equal to that received from the lower half. This "balancing of the beam" was continued until the two halves agreed within one or two per cent. A failure to do this would tend to displace the positions of the maxima and minima in the curves.

The Boltzmann mirrors were set for no difference of path, the deflections on the main and on the check receiver were read and the ratio of main to check recorded. The mirrors were then separated by some amount and the ratio of main to check again recorded; then the mirrors set for zero difference of path and the ratio of main to check recorded. The second ratio was divided by the average of the first and third, expressed in per cent. and taken as one reading for the given difference of path. This was repeated for any point as often as necessary to be reasonably certain of the value. In this way readings were taken for as many different settings of the Boltzmann mirrors as desired, the values always being expressed in per cent. of energy received for zero

difference of path. A curve was plotted with distance between Boltzmann mirrors as abscissæ and per cent. of energy as ordinates, from which the wave-length was then determined, it being twice the average distance between successive maxima or successive minima.

There is always a certain amount of energy, which arrives at the receiver by diffuse reflection from the walls of the room, which we may call "stray energy." It is small compared with the energy in the reflected beam and is constant in amount regardless of the setting of the interference apparatus. It will therefore not affect the distance between maxima and minima. This stray quantity must, however, always be subtracted before plotting curves which are to be used in studying the damping of the oscillation.

WAVE-LENGTH INTERFERENCE CURVES.

With S_1 and S_2 total reflectors the wave-length measured is that of the oscillating system itself. Figures 5, *A*, and 6, *A*, are typical curves obtained under these conditions. The wave-length being in each case about 76 mm., which could be determined within two per cent. The maxima and minima are fairly evenly spaced (see Fig. 6, *A*) with the first minimum about 12 per cent. of the total energy, the first maximum 68 per cent., the second minimum 33 per cent. and the second maximum 47 per cent. of the total energy at zero difference of path. For wider separations of the Boltzmann mirrors the periodic interference ceased and the energy received was nearly constant. That is to say, when the distance between the mirrors is one wave-length, or when the total difference of path is two wave-lengths, interference no longer takes place because of the high damping. If the curve is carried out farther it is seen to lie along the 45 per cent. axis. It was found for these curves that the value for this axis was in every case approximately equal to the sum of the percentages of energy received from the two halves of the mirror separately. Each half would accordingly produce an effect of about 23 per cent. If the two effects are impressed on the receiver in such time relations as not to produce interference of the oscillation in the receiver then the total energy is simply additive. But if they be added so as to produce double the amplitude, the energy being proportional to the square of the amplitude, the deflections will be four times as great as those produced by each half alone. This is the reinforcement obtained when the two mirrors are set for zero difference of path. The fact that the axis of symmetry is not the 50 per cent. axis may be due to diffraction effects produced by the adjacent edges of the two halves of the Boltzmann mirrors.

Fig. 5, *B*, shows the results when S_1 was made a screen of resonators and S_2 was a total reflector, the oscillator being the same as for 5, *A* ($\lambda = 76$ mm.). S_1 acted as a selective screen for the highly damped energy falling upon it and reflected a new wave-length determined more by the dimensions and distribution of the resonators on the screen than by the incident wave-length. The wave-length was increased to 99 mm. and the damping considerably diminished, as can be best seen by comparing the second and third maxima and minima in the two curves *A* and *B*.

Fig. 5, *C*, was the result when S_2 was made a screen of resonators

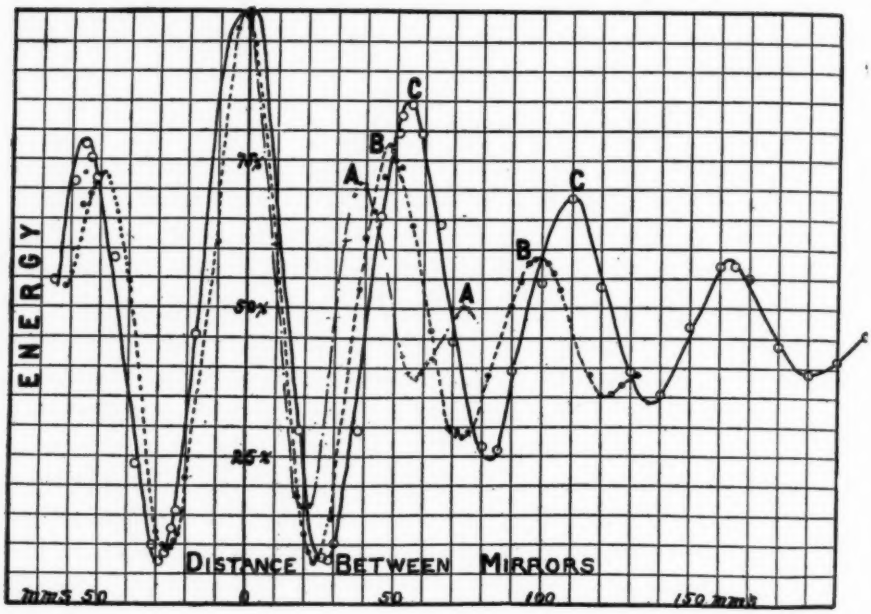


Fig. 5.

Interference curves.

exactly similar to S_1, the vibrator being the same as before. S_2 was being excited by energy which was the result of a combination of vibrator and one resonator screen. The damping was again reduced and the wave-length increased to 110 mm. with the maxima and minima at very evenly spaced intervals. This was the wave-length due to the natural frequency of the resonators themselves, as was later shown by tuning the vibrator to the resonators and measuring the resultant wave-length. Starting with a highly damped radiation from the oscillating system of 76 mm. wave-length we have obtained after two reflections from resonating screens a wave of 110 mm. in which the damping is surprisingly small. If we regard the highly damped radiation from the oscillator as a con-

tinuous spectrum then the analogy to the case of *reststrahlen* is very striking.

It is to be expected that the purest or least damped radiation will be obtained, with but two reflections, if the screens and vibrator are very nearly in tune. These conditions were obtained with two screens made of resonators 36 mm. long and 0.3 mm. wide. The resonators were mere threads of tinfoil with an open space of 54 mm. between them from side to side and 10 mm. between them from end to end. The exciting source, the vibrator, was sending out a wave of 76 mm. length while the wave-length after two reflections was only 84 mm., so that the *second* screen could be considered excited by energy in tune with its resonators. Fig. 6, *B*, shows the results of these conditions. They meet the expectation.

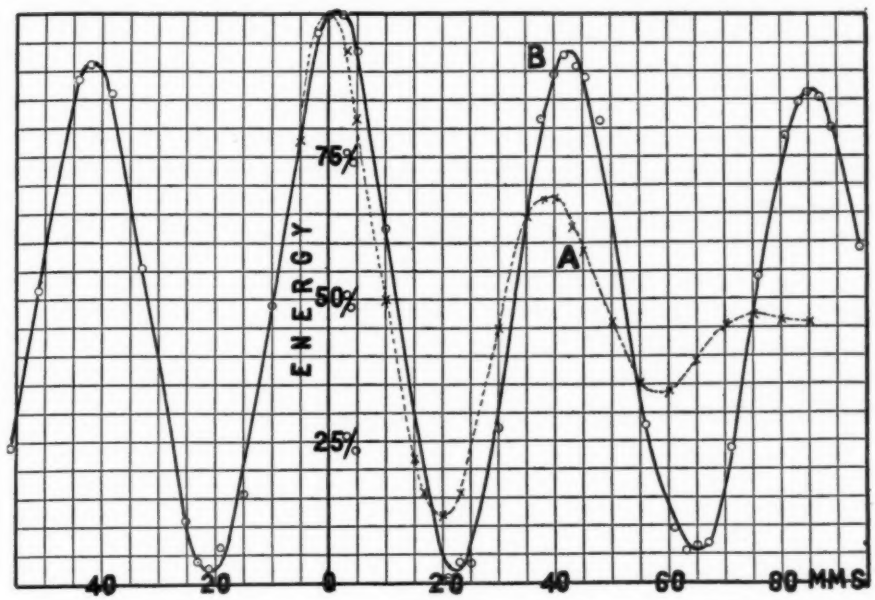


Fig. 6.

Interference curves.

The oscillator curve and the curve after two reflections are plotted together in Fig. 6 for purposes of comparison. The energy at the first minimum in *A* is 12 per cent. while that in *B* is only 2 per cent.; at the first maximum in the one it is 68 per cent. and in the other 93 per cent., showing the much sharper interference obtained in the case of the resonator radiation. The stray energy is so large a part of the total energy received when the mirrors are set for greatest interference that the exact value of the energy at the minimum of the resonator curve cannot be

determined. One of the points of this interference curve which has been corrected for stray energy will be seen to lie almost on the zero energy line.

With a double reflection, then, from linear resonators in tune with the oscillator, energy of very little damping and of very definite wave-length is to be obtained. The properties of the resonator radiation are those of a "monoperiodic" oscillation to a much higher degree than the radiation of the oscillator. These screens may therefore be used to advantage as a *new source* in investigations with short electric waves.

Reflection Coefficients.

While the smallness of the damping is about all that could be desired for certain uses of the electrical oscillation, yet difficulties arise due to the lack of energy at one's disposal. A large part of the incident energy passes on through the screen and must be reflected out of the beam by a deflecting mirror located a short distance behind the resonators. The amount of energy transmitted depends on the number of resonators in the field and the closeness of their tuning to the energy which falls upon them. A numerical study of these quantities in a particular case will be of interest.

The screens used for this test were those used to obtain the curve in Fig. 6, *B*. The resonators were 36 mm. long and 0.3 mm. wide with an open space of 54 mm. between the horizontal rows. There were seventeen columns with one cm. between the ends of resonators. Their natural wave-length was 84 mm. In Table I. the first column gives the wave-length of the highly damped oscillating system.

Table I.

λ, Mm.	$\frac{e_1}{E}$, Per Cent.	$\frac{e_2}{e_1}$, Per Cent.
75	14	36
85	16	34
95	15	36
120	13	31

E is the energy arriving at the receiver when S_1 and S_2 (Fig. 3) are total reflectors; e_1 is the energy when S_1 is one of the above-mentioned screens of resonators and S_2 is a total reflector; e_2 is the energy received when S_1 and S_2 are both screens of resonators. $e_1/E \cdot 100$ gives the per cent. of incident energy reflected by the first screen and $e_2/e_1 \cdot 100$ gives the percentage reflection of the second screen. It will be seen then that under the best conditions of tuning the energy after reflection from two

resonator screens was only 34 per cent. of 16 per cent., or 5.4 per cent. of the initial energy from the oscillator when two total reflectors were used. The above resonator distribution was rather sparse and the energy might be greatly increased by increasing the number of resonators.

A comparison of the two columns e_1/E and e_2/e_1 is also of interest as a further evidence of the "purity" of resonator radiation. Energy relations of this kind are difficult to establish to any high degree of precision, but the table shows that in every case the ratio e_2/e_1 was more than twice as great as e_1/E. Even for the closest tuning the first screen reflected only 16 per cent. of the highly damped radiation falling upon it whereas the second screen, which was excited by the energy coming from the first screen, reflected 34 per cent. of the incident energy.

This increase in the reflection coefficient is not due to increased polarization of the oscillations, as the oscillator radiation has been shown to be almost entirely polarized. This can also be seen from Table II.

TABLE II.

λ, Mm.	$\frac{e_1}{E}$, Per Cent.	$\frac{e_2}{e_1}$, Per Cent.
75	10	14
85	12	16
95	16	22
120	17	23

For this test the screens consisted of continuous lengths of foil across the frame parallel to the electric vector. They were non-selective reflection gratings with the same distance between the lines, *i. e.*, 54 mm., as between the resonators of the former screens. The numbers in the two columns of Table II. increase about proportionately due to the fact that, as the incident wave-length increases, the amount reflected increases for any given open space in the grids. This is in accord with the results of Blake and Fountain.[1] They found that a wire grid would act as a total reflector if the distance between wires was as small as one fortieth of the incident wave-length.

DAMPING COEFFICIENTS.

Klemenčič and Czermak[2] have worked out an expression for the damping coefficient which can be applied to the interference curves obtained with the Boltzmann mirrors. By integrating and adding the effects

[1] Blake and Fountain, PHYS. REV., Vol. XXIII., 1906, pp. 274–276.

[2] Klemenčič u. Czermak, Annalen d. Physik u. Chemie, 1893, Vol. 50, p. 179.

produced by the two halves of the mirror they obtain the final form

$$\alpha = \frac{A^2 T}{4\gamma}(1 - e^{-2\gamma x/\lambda} + C^2),$$

in which γ is the damping coefficient, T the period of the oscillation, x the difference in path of the two halves of the beam, λ the wave-length, A is a constant and C an expression involving x, γ and λ. α is proportional to the galvanometer deflection. Values of α are found when $x = 0$, $x = \lambda/2$, $x = \lambda$, $x = n\lambda/2$. If these values be called α_1, α_2, α_3 and α_n we get the expression

$$\frac{\alpha_1 - \alpha_2}{\alpha_3 - \alpha_2} = e^{\gamma/2},$$

whence

$$\gamma = 2 \text{ nat. log.} \frac{\alpha_1 - \alpha_2}{\alpha_3 - \alpha_2}.$$

This equation is in a form which can be applied to any two maxima and the minimum between them in an interference curve.

The conditions for which the above formula was developed were that the difference in path be small compared with the distance from the interference mirrors to the source, that the wave be a plane wave, and that the damping coefficient be small. Hull[1] showed later that the receiver affected the wave-length curves and that if the above formula is to be used the receiver must be dead beat or non-selective. All of these conditions have been met in the present work and the formula was used for the wave-length curves obtained. When applied to the curve showing least damping (Fig. 6, *B*) the value for γ comes out about 0.15, which is quite small and in good agreement with the values obtained by theoretical considerations of various writers. (For a general review of these results the reader is referred to J. A. Fleming's Principles of Electric Wave Telegraphy, pp. 189 to 198.) A coefficient of 0.15 indicates that the energy dies down to one per cent. in about fifteen complete oscillations. The same equation applied to the oscillator curve gives a value of about 1.4 for γ, which is in good agreement with Webb's[2] values. According to this coefficient the value of the energy falls to one per cent. after about two complete oscillations, which gives a good idea of the great difference in the damping of the energy from the two sources. The resonator damping is probably almost wholly due to radiation. It may be of interest to note that the ohmic resistance of the resonators to direct current is not more than five or six ohms in the extreme cases.

[1] G. F. Hull, PHYS. REV., 1897, Vol. 5, p. 231.

[2] Webb and Woodman, PHYS. REV., 1909, Vol. XXIX., p. 116.

TUNING CURVES AND OVERTONES.

The properties of this resonator radiation are also brought out in tuning curves. The second screen of resonators (S_2) was varied while the oscillator and S_1 were kept constant. Balls of 7/8 inch diameter were used for the vibrator, the wave-length of which was very nearly the same as that of S_1, which consisted of resonators 2×60 mm. with a lateral separation of 42 mm. A set of screens with the resonators varying in length from 36 to 140 mm. was used in turn at S_2. These resonators were all placed on the screens with the same separation both laterally and end to end so that there was practically the same amount of metal in the field for all screens although the number of resonators on the various screens differed.

The result is seen in Fig. 7, *A*, where the ordinates are the energy and

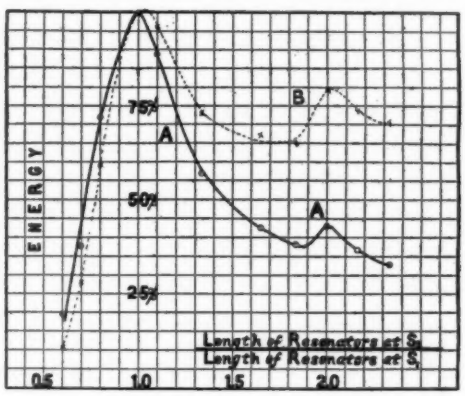


Fig. 7.

Resonator tuning curve.

the abscissæ the ratio of the length of resonators at S_2 to the length at S_1. The energy received at the main receiver was taken as 100 per cent. when S_2 was a screen exactly like S_1, and the energy for other screens at S_2 was referred to this value. There is a very decided maximum for the ratio unity, *i. e.*, when the two screens are exactly alike, and another maximum, though much smaller, when the resonators at S_2 are just twice as long as those at S_1.

This shows then, as was to be expected, that the maximum reflection is to be obtained when the excited screen is exactly in tune with the exciting source. It furthermore shows, what has not been heretofore conclusively shown experimentally (so far as the authors are aware) that a linear resonator may oscillate in an overtone with a wave-length equal to half its fundamental wave-length. There was resonance when the

second screen was excited by a wave-length equal to half its fundamental. Furthermore the wave-length measured under this condition of resonance was found to be that of the overtone and not that of the fundamental. The damping and equal spacing of maxima and minima in the wave-length curve were similar to that obtained with resonators oscillating in their fundamental frequency.

This result is in agreement with some electromagnetic experiments performed by Rubens and Nichols[1] with heat radiation of great wave-length. Metallic resonators ruled on silvered glass were used as reflecting surfaces for heat waves of about 22.5 μ. Reflection maxima were found for lengths of 12.4 μ and again for 24.4 μ. Interpreted in the light of our Fig. 7 one would say the 12.4 μ resonators were oscillating in their fundamental frequency whereas the 24.4 μ were oscillating with the frequency of their first overtone. It is of interest to notice furthermore that they found even greater reflection when the surfaces were ruled into continuous lengths across the plate, *i. e.*, when the resonators were practically of infinite length compared to the wave-length falling upon them. In this case the plates were probably acting as non-selective reflection grids and the percentage reflection depended on the relation of the distance between the metallic strips to the incident wave-length. It was hardly a resonance phenomenon. If the strips had been close enough together the surface would have acted as a total reflector.

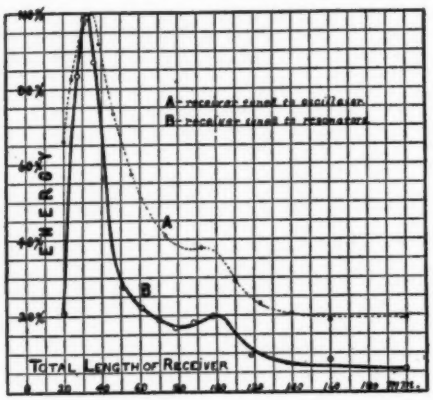


Fig. 8.

Receiver tuning curves.

We are at a loss to understand why Blake and Fountain[2] were not able to obtain this first overtone resonance in their reflection and transmission curves. There may have been some difficulties due to the fact

[1] H. Rubens and E. F. Nichols, PHYS. REV., 1897, Vol. V., p. 164.

[2] L. c., pp. 269–273.

that a selective receiver was used in all of their work. It should be mentioned that their method of tuning differed from the present method in that they started with long resonators and clipped off successive amounts; thus keeping the total number of resonators in the field constant, whereas in our work separate screens were made up for each length of resonator and enough resonators placed on the screen to maintain both lateral and end to end separation the same. The former method will surely bring about differences in energy due to great differences in the amount of surface covered by resonators. This would make the resonance to the fundamental much less striking but should not greatly affect the position of the maximum nor the appearance of the overtone resonance. Fig. 7, *B*, shows what should be expected in this case, *i. e.*, when the number of resonators on all screens was kept constant. This curve was obtained from Fig. 7, *A*, on the assumption that the reflected energy is directly proportional to the number of resonators in the field.

Receiver Tuning Curves.—The relatively small damping or increased

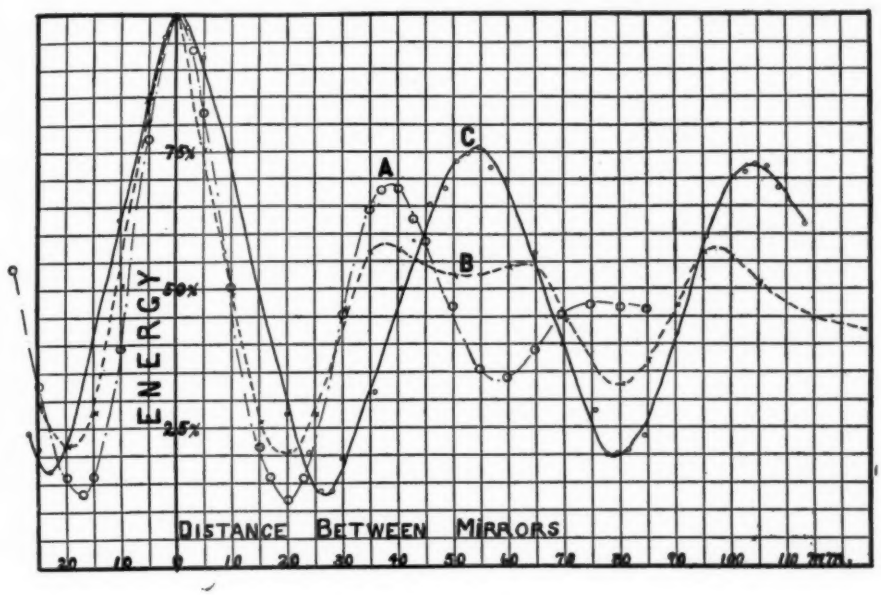


Fig. 9.

Interference curves.

"purity" of the resonator radiation was also seen when the Klemenčič receiver itself was tuned to the oscillator directly, and when tuned to the resonator radiation. The curves were obtained in the same way as those of Webb and Woodman. The galvanometer deflection was first taken when the receiver was long and again for each length as pieces were clipped

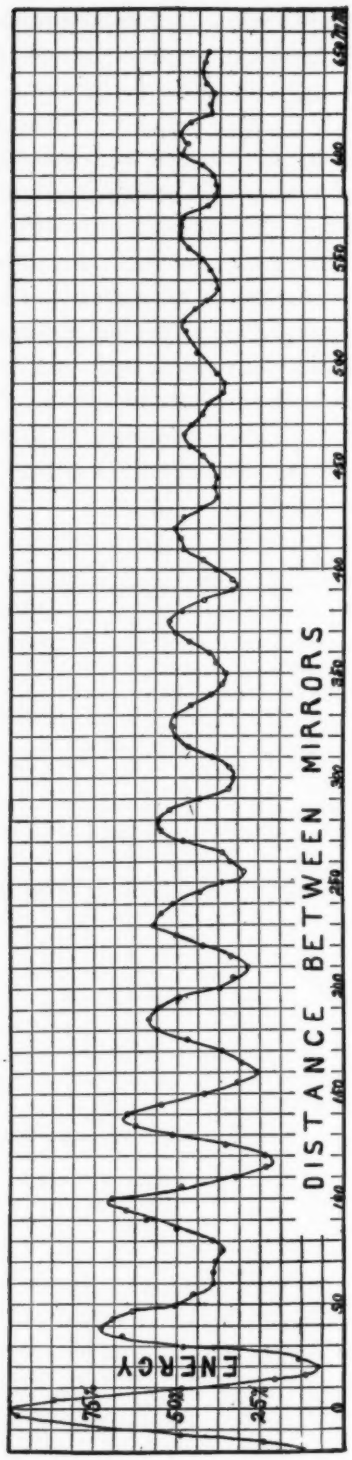


Fig. 10. Interference curve.

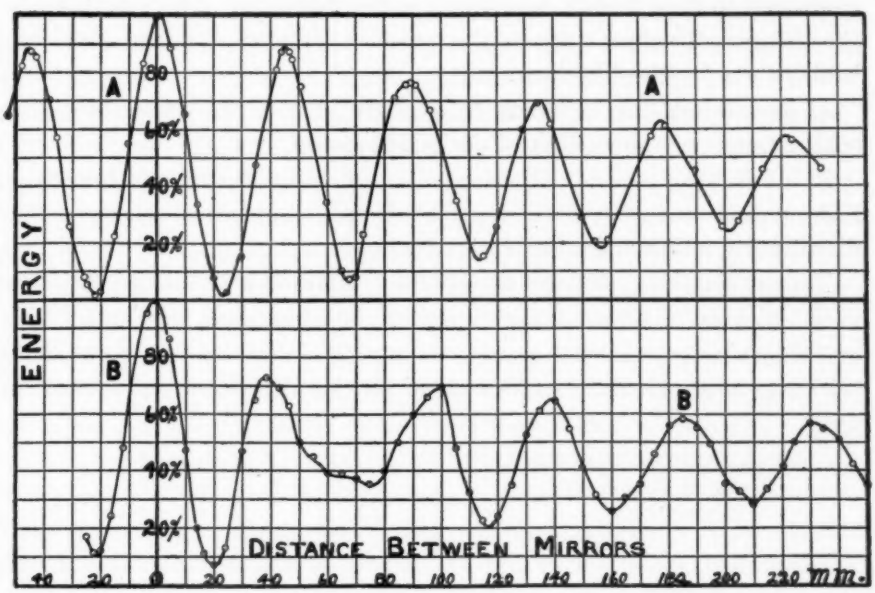
from the receiver wings. The results are shown in Fig. 8. Curve *A* shows the tuning of the receiver to the energy of the oscillator and curve *B* the tuning to the energy received from two reflections of resonator screens. The energy for the tuned length is in each case taken as 100 per cent. and the other values referred to this, and the percentages plotted as ordinates with the corresponding total receiver length in millimeters as abscissæ. These curves as is usual for receiver tuning curves show the overtone at approximately three times the fundamental length. The first overtone cannot appear in the receiver as it causes a node of current at the center where the thermo-junction is located. That the first overtone does exist in a resonator was shown in the preceding section.

The average value of the ratio of wave-length of resonators to the fundamental resonance length of receiver was found to be 2.46 (2.39, 2.43, 2.48, 2.50, 2.50). It must be stated in this connection that for mechanical reasons the base of the receiver used for this tuning work could not be made exactly the same as that used in the non-selective receiver for wave-length determinations. This fact may have something to do with the exact value of ratio obtained above.

Of this ratio more will be said in a following paper.

The subject of overtones will be further mentioned in a paper in this issue on the investigations of the effect of the dimensions of the resonators, particularly the width, on their resonance qualities. A comparison of Fig. 5 with Fig. 9 will, for the present, serve to show this effect. In each figure *A* is the vibrator curve, *B* is the curve from one screen and *C* is the curve from two screens of resonators. In Fig. 5 the resonators are linear while in Fig. 9 they are square. The vibrator curve is the same in the two cases and the *C* curves are seen to be of the same wave-length, but the *B* curves in the two cases are quite different. *B* in Fig. 9 shows a very marked tendency to break down into an overtone, or at least gives a complex radiation due to the fact that resonators and oscillator are not in close enough tune. It is important to note that the second screen makes use of this rather complex source and is caused to oscillate with its own natural frequency.

Fig. 11, *B*, is a further illustration of the effect of vibrator and resonators



Interference curves.

Fig. 11.

not being in close enough tune. The maxima and minima are not evenly spaced, nevertheless the oscillation is maintained for about fifteen wave-lengths as can be seen in Fig. 10. Fig. 11, *A*, shows a typical wave-length curve when screens and oscillator are in tune with each other.

If this form of apparatus is to be used as a source of short, undamped electric waves, then it is evident, the closer the tuning between oscillator and resonator screen the better. If two reflections can be had then the exciting wave may differ from the natural wave-length of the resonators by as much as 20 or 30 per cent., while if only one reflection can be had this difference should not be more than 10 or 15 per cent.

SUMMARY.

1. The results of multiple reflection of electric waves from selective resonator screens are very similar to those of multiple or selective reflection in the infra-red.

2. Multiple reflection affords a practicable and easy method of obtaining short electric waves having a very definite period, less damped than the oscillator and free from "mirror action" and other disturbing influences.

3. The damping coefficient for resonators, whose width was negligible compared to their length, was found to be about 0.15, which agrees with the theoretical value, whereas the experimentally determined coefficient is about 1.4 for the Righi vibrator.

4. Resonators can be made to oscillate in their first overtone, *i. e.*, with a frequency twice that of the fundamental.

In a following paper is discussed an application of this multiple reflection method to a systematic study of the resonators themselves and the distribution required for any desired wave-length.

The authors take pleasure in expressing here their thanks to President E. F. Nichols, at whose suggestion these investigations were undertaken and they wish especially to express their appreciation for the many helpful criticisms and valuable suggestions of Dr. H. W. Webb throughout the work.

PHŒNIX PHYSICAL LABORATORY,
COLUMBIA UNIVERSITY,
March, 1913.

A SYSTEMATIC STUDY OF LINEAR AND NON-LINEAR RESONATORS FOR SHORT ELECTRIC WAVES.

BY W. S. NELMS AND W. L. SEVERINGHAUS.

INTRODUCTION.

IN the preceding paper the so-called "multiple reflection method" for investigating the radiation from screens of resonators has been discussed. The results and advantages of this method have been given there. It was shown that multiple reflection gave a radiation which was little damped and contains a single wave-length. With this kind of radiation we have determined the relation of the dimensions and the distribution of the resonators to the wave-length, damping, and amount of energy radiated from a screen. The question as to how wide the resonator may be and still be considered linear has also been investigated, in as much as the term "linear resonator" has been used rather loosely by other investigators.

The apparatus described in the first paper was used again in this investigation. There was a primary oscillating system (*source*), a reflecting system, and a receiving system. The oscillating system, or primary source of energy, consisted of a Righi vibrator placed at the focus of a spherical parabolic mirror. The reflecting system was the screens of tinfoil resonators to be studied together with a Boltzmann mirror for measuring the wave-lengths. The receiving system had a non-selective receiver of the Klemenčič type at the focus of a parabolic mirror similar to that used in the oscillating system. There was also a non-selective check receiver, the use of which was explained in the previous paper. The dimensions and details of the apparatus, as well as the arrangement of the parts, are given at length in the same place. Throughout the present investigation the apparatus was kept as nearly as possibly the same. It was found best to keep the period of the primary source as nearly as possible the same as the natural period of the resonators under investigation. This was accomplished by changing the vibrator from time to time.

A change in the dimensions of the resonators must be considered as due to a variation of the length, width or thickness: and by a change in

the distribution is to be understood a change in the distance between consecutive resonators measured parallel to the electric component or a change in the distance measured perpendicular to this. The thickness of all the resonators was kept small and constant. In the discussion which follows, the dimension of a resonator measured parallel to the electric component of the oscillation in the incident energy is called the length, the dimension perpendicular to this is called the width. In general, the length was greater than the width but in some cases the width was

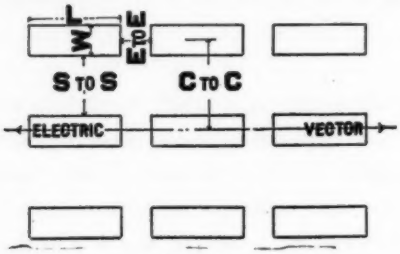


Fig. 1.

C to *C*, center-to-center distance; *S* to *S*, side-to-side distance; *E* to *E*, end-to-end distance; *W*, width; *L*, length.

several times as great as the length. Fig. 1 will make clear the meaning of the terms to be used.

The length of a resonator is the most important factor in determining its natural frequency. It seems well established that the wave-length reradiated is directly proportional to the length of the linear resonator, if the separation be kept in a constant ratio to the length of the resonator. The relation may be expressed by an equation of the form $\lambda = KL$, where K is a constant whose value depends on other factors than the length. The value of this constant given by theoretical and experimental investigators[1] seems to lie between 2 and 2.6. The theoretical values apply to the case in which there is but one resonator in the field. The primary object of this investigation was to find the value of K for any distribution of resonators for which the corresponding energy is sufficiently large to permit of the application of the present apparatus and method.

The results of this investigation are presented under three headings, viz, the effect on the period of oscillation of the resonators brought about by a change in

I. The distance between the resonators measured *parallel* to the electric component in the energy,

[1] MacDonald, Electric Waves, p. 111; Poincare, Les Oscillations Electric; M. Abraham, Wied. Ann., 1898, 66, p. 435; Blake and Fountain, PHYS. REV., 1906, Vol. XXIII., p. 276; Webb and Woodman, PHYS. REV., 1909, Vol. XXIX., p. 131; Ives, PHYS. REV., 1910, Vol. XXX., p. 199.

II. The distance between the resonators measured *perpendicular* to the electric component in the energy,

III. The *width* of the resonator.

I. THE EFFECT OF A CHANGE IN THE DISTANCE BETWEEN THE RESONATORS MEASURED PARALLEL TO THE ELECTRIC OSCILLATION.

It was not expected that the distance between the ends of the resonators (*E* to *E*, Fig. 1) would affect their period of oscillation. This proved to be true for linear resonators. There is but little mutual capacity effect for such small surfaces, and there is no interlinking of the field of one with the circuit of the other, hence no mutual induction effect. Even with very wide resonators when the distance between the ends of the resonators was changed experiments showed that there was no effect on the period of oscillation of the resonator.

For the present "linear resonators" shall be considered as those having a width not greater than *one tenth* of the length. Later it will appear that a "linear resonator" may have a width as great as one fourth of the length. Table I. shows the results due to end-to-end variations. Using linear resonators, this end to end distance between resonators was varied from 4 mm. to 22 mm. The length of the resonators was changed from 23 mm. to 55 mm. The changes in wave-lengths noted were those due to changes in length of the resonators and changes in the distance between the resonators in the direction perpendicular to the electric oscillation. The fourth column of the table (Measured Wave-lengths) gives the wave-lengths measured by the Boltzmann mirror for the different screens of resonators; the fifth column gives the wave-length calculated for screens of resonators having the same dimensions and center-to-center distance, but having an end-to-end distance of 10 mm. How these calculated wave-lengths were obtained will appear in the next section, which relates to the effect of varying the center-to-center distance. The method of measuring the wave-lengths is accurate to about two per cent.; this is about the maximum variation of any measured wave-length from that calculated for the similar screen with a different end-to-end distance. The results in Table I. must be considered for each screen separately, for the separation center-to-center was not the same for the different cases, and screens having one separation cannot be compared with those having some other separation. With the resonators 226 and 590 mm. wide only one row of resonators was put on the screens, hence the different screens may be compared directly.

The fact that there was no end-to-end effect for the linear resonators

TABLE I.

Width, Mm.	Length, Mm.	End-to-end Distance, Mm.	Wave-length.	
			Measured, Mm.	Calculated for E to E 10 Mm., Mm.
2	23	22	58	57
2	36	10	95	96
2	36	15	97	96
2	50	4	113	113
2	55	5	110	113
3	36	10	83.5	85
3	36	15	95	97
18	36	10	91	91
18	36	20	81	81
18	36	15	100	97
226	36	10	72.3	
226	36	56	72	
590	50	43	110	
590	50	15	112	
590	50	20	110	

does not require that there be no such effect for the wide resonators. It was, however, demonstrated that there was no effect of this kind even for the very wide resonators. As appears in Table I., resonators varying in width from 18 mm. to 590 mm. (that is, from one half to twelve times the length) gave little end-to-end effect. With resonators 18 mm. wide there was no change in the wave-length when this end-to-end distance was changed from 10 mm. to 20 mm.; resonators 226 mm. wide showed no change in wave-length for a variation of this distance from 10 mm. to 56 mm. The end-to-end distance was changed from 15 to 43 mm. with the very wide resonators (590 mm. wide), the change in wave-length was less than two per cent.

This work on the effect of a change in the distance between the resonators measured parallel to the electric oscillation confirms the work of Blake and Fountain[1] and extends it to apply to the case of non-linear resonators. We find then that there is no effect on the wave-length of the radiation from a screen of resonators when the resonators are moved apart in the direction of the electric component of the oscillation.

II. THE EFFECT OF VARYING THE DISTANCE BETWEEN THE RESONATORS MEASURED PERPENDICULAR TO THE ELECTRIC COMPONENT OF THE OSCILLATION IN THE RESONATORS.

A change in the distance between adjoining resonators measured perpendicular to the electric oscillation produces a change in the wave-

[1] Blake and Fountain, PHYS. REV., 1906, Vol. XXIII., p. 268.

length of the energy radiated by a set of resonators of given length. Table II. and Fig. 2, *A*, give the results of this investigation on the effect of this change of distance for the linear resonators (resonators 36 mm. long). The curve represents the relation between the wave-length and the center-to-center separation for these linear resonators. There is a minimum value for the wave-length when the separation has the value of 9 mm., *i. e.*, the distance between the resonators is 0.4 of the length of the resonators. From this point the curve is practically a straight line, the wave-length increasing as the separation is increased. This straight line relation cannot hold indefinitely, but apperas to hold past the point for which the separation has the value of 90 mm. or 2.5 times the length of the resonators. The separation represented by this value of 90 mm. is the greatest for which the multiple reflection methods could be used. For separations greater than this the energy became very small because of the few resonators in the field, and the method was then unreliable.

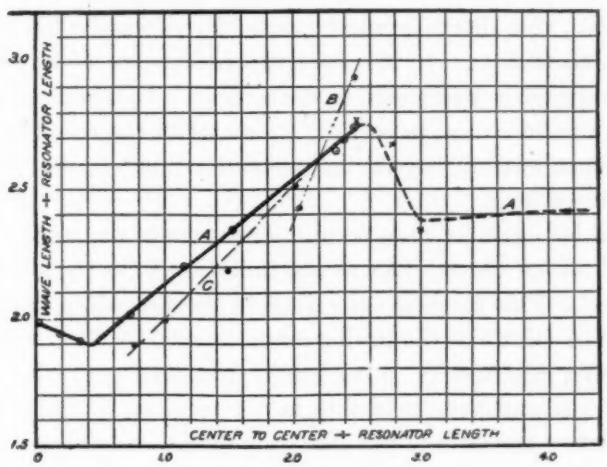


Fig. 2.

⊙, points taken by "multiple reflection"; ×, points taken by tuning method; ●, non-linear resonators.

The abscissæ of the curve as it is plotted in Fig. 2, *A*, were taken as the ratios of the distance between the resonators to the length of the resonator, and the ordinates as the ratios of the wave-length to the resonator length. These coördinates were chosen in order to make the curve more general in its application. With these coördinates the curve can be applied to any resonator system regardless of the length of the resonators. For with any two sets of resonators both variables are divided by the same factor, and the shape of the curve is maintained.

TABLE II.

C to C Distance, Mm.	Wave-length Measured, Mm.	Wave-length / Reso. Length	C to C / Reso. Length	Wave-length of Source, Mm.
7	69	1.94	0.194	76
12	68	1.91	0.333	76
26	73	2.02	0.723	76
42	79.5	2.20	1.166	76
56	84.6	2.35	1.555	76
82	95	2.63	2.380	76
86.5	99	2.69	2.400	87
90	97	2.72	2.500	87
	All resonators 2 mm. wide and 36 mm. long.			

The data contained in Table II. were obtained by using the same linear resonators throughout, keeping the same distance between the ends of the resonators, and varying only the distance between the resonators in the direction perpendicular to the electric oscillation. As will be seen from the table, this distance was changed by successive small amounts from 7 mm. to 90 mm. In all cases two reflections were used and the source was kept fairly well tuned to the screens. Under these conditions, as shown in the preceding paper, the character of the resulting radiation depends almost entirely upon the resonators and is practically independent of the source.

For separations greater than 90 mm. or 2.5 times the resonator length, a method of tuning the receiver to the radiation from the screens had to be introduced. This method (described in the preceding paper) is open to the same objections[1] as the tuning methods used by other investigators. In any tuning method of wave-length measurement, the maximum energy may not and very probably is not received when the tuning is exact, *i. e.*, when the system has the same period as the incident energy. There is in all cases a decrease in the energy due to the decrease in the size of the resonator or receiver, as the case may be. This will tend to shift the length taken as the tuning length toward the large values and give an incorrect value for the wave-length. This objection may be brought against that part of the curve in Fig. 2, *A*, which lies beyond the abscissa 2.5. It may also be brought against the work of all investigators who have used a tuning method for studying resonators of this type.

Although the method of study is open to the above objections, it was adopted in the present investigations since it was very desirable to carry the curve beyond the value of 2.5 for the abscissa. From the part of the curve beyond this point it is possible to estimate the value of the

[1] Webb and Woodman, PHYS. REV., Aug., 1909, Vol. XXIX.

ordinate of the curve when the resonators are separated to such a distance that they no longer influence the period of one another. The value of the constant K connecting the resonator length and the wave-length when there is only one resonator in the field would thus be found. The exact shape of the curve beyond the point for which the abscissa is 2.5 has not been accurately determined. However, it appears to turn at about this point and to approach the ordinate having the value of about 2.4 as its value when the resonators no longer affect each other. This then would indicate the ratio of the wave-length to the resonator length when there is but one resonator in the field. The exact value of K is still in doubt, but the most probable value does not agree with the theoretical value given by Abraham.[1] The value does agree very well with that obtained by other investigators. For a radiating system the ratio of the wave-length to the resonator length seems to be greater than two.[2]

TABLE III.

C to C Dist., Mm.	Wave-length Measured, Mm.	Wave-length Calculated, Mm.	$\frac{C \text{ to } C}{\text{Reso. Length}}$	Length of Resonator, Mm.	E to E Dist., Mm.	Wave-length of Source, Mm.
42	58	57	1.825	23	22	72
42	58.5	57	1.825	23	22	76
42	91	90	1.000	42	10	87
42	94	90	1.000	42	10	118
67	113	113	1.340	50	4	87
42	111	113	0.722	55	5	76
84	116	120	1.400	60	10	118
42	119	122	0.700	60	10	118

As a test of the accuracy of the curve showing the relation of the separation to the wave-length emitted, a number of screens were made and the corresponding wave-lengths measured and compared with those calculated from the curve. The results are shown in Table III. In one column are given the measured values of the wave-lengths and in a parallel column the calculated values. Screens of resonators varying in length from 23 mm. to 60 mm. and having a C to C separation varying from 40 to 84 mm. were tested. The distance between the ends of the resonators are also given, but as shown in the last section this factor does not affect the wave-length. Linear resonators only were used, the width being 2 mm. in all cases. A comparison of the two columns mentioned above shows that in no case is the variation of the calculated value for the wave-length from the measured value greater than about 3 per cent. over a range of wave-lengths varying from 58 mm. to 119 mm.

[1] M. Abraham, Wied. Ann. d. Phys., 1898, 66, p. 435.

[2] Pierce's Principles of Wireless Telegraphy, p. 116.

Not only does this curve represent the results for the apparatus used in the present case, but the results of other investigators using other forms of apparatus agree very well with this curve. Aschkinass and Schaefer,[1] in 1901, published the results of some work done to determine the dielectric constants of certain liquids. Their investigation was made by studying the effect of the liquid on the period of resonators of the same type as ours when immersed in the liquid. The resonators had the dimensions and distribution given below:

Length of resonator.....45 mm.
Separation perpendicular to length.....27 mm.
Width of resonators....."linear"
Wave-length in tune with resonators *in air*.....90 mm.

For a screen of this description the curve Fig. 2, *A*, calls for a wave-length of 90 mm.

Woodman and Webb[2] found that resonators 29 mm. long, with a separation of 30 mm., were in tune with a source having a "wave-length of 62 to 64 mm." The curve shows that a screen with resonators of these dimensions should have a natural frequency corresponding to a wave-length of 62.5 mm. The wave-lengths given by the various observers mentioned above were measured by different methods, and differing from that used in obtaining the curve, so that the agreement between their results and the calculated values cannot in any way be due to a similarity of apparatus or methods. Hence it is evident that the curve given in Fig. 2, *A*, which represents the effect of the separation on the wave-length expresses very closely the correct relation between these two quantities.

Blake and Fountain[3] in a paper relating to linear resonators, give the following data for three of their screens:

No.	Resonator Length, Mm.	Separation, Mm.
1	53	30
2	49	60
3	44	100

The curve obtained in the present investigation gives the following values for the wave-lengths from these screens:

No.	Wave-length.
1	104
2	109
3	115

[1] Aschkinass and Schaefer, Ann. d. Phys. 1901, 5, p. 489.
[2] Woodman and Webb, PHYS. REV., Vol. XXX., 1910, p. 575.
[3] Blake and Fountain, PHYS. REV., Oct., 1906, Vol. XXIII.

The investigators state that these screens were in tune with a vibrator having a wave-length of 99 mm.[1] Our study of their interference curve for the wave-length of their source seems to indicate that the wave-length may have been about 108 mm. Webb and Woodman[2] have shown that the tuning method is not very reliable for wave-length measurements and that the tuned receiver is unsatisfactory for this kind of work. The results given for these screens do not agree as well with the calculated results from the curve as those of the other investigators mentioned. But the errors to be expected from the tuning method are more than sufficient to account for the discrepancy.

In the same paper (page 268) Blake and Fountain have given a curve representing the relation of the separation to the wave-length for the screen. Their curve does not seem to fit the facts as found in the present investigation. The discrepancy is one of amount and not of kind.

The curve, Fig. 2, *A*, applies to linear resonators only. If the multiple reflection method is to be of value, we must be able to determine beforehand what the frequency of the resonators will be on any given screen. Hence the effect of the separation on the wave-length for the *non-linear* resonators must also be known. The effects for square resonators (36 mm. long and 36 mm. wide) and for resonators 36 mm. long and 18 mm. wide are shown in Fig. 2, *B*, and 2, *C*. It is seen that the curves are similar to that for the linear resonators but have a greater slope. Hence for wide resonators the wave-length increases by a greater amount for the same increase in separation than for the linear resonators. For the intermediate widths the slope of the curve is also intermediate.

When the wide resonators are used a question arises as to how this separation is to be measured. Shall it be taken as the distance between the adjacent edges of the resonators or shall it be taken as the distance between corresponding points in the resonators, as for example between the centers (*C* to *C*)? A study of this point shows that the results are more simply correlated if the separation be taken as the distance between the centers of the resonators. The reason for this conclusion will be taken up more fully in the next section.

The curves in Fig. 2 were used for determining the length and distribution of resonators on screens to give desired wave-lengths. The length chosen depended on the energy required, for the energy reradiated decreased with a decrease in the length of the resonator as well as with the separation. A very satisfactory condition was found when the length of the resonator was approximately one half of the wave-length desired.

[1] Blake and Fountain, PHYS. REV., 1906, Vol. XXIII., pp. 263–4.

[2] Webb and Woodman, l. c., p. 123.

Having chosen the length of the resonator the exact distribution to be used was determined from the curve. By taking the ratio of the length of the wave to the length of the resonator as the ordinate, the ratio of the separation to the length of the resonator was the abscissa of the point on the curve. The distance between the ends of the resonators (E to E) does not affect the wave-length, hence this distance was made 10 or 15 mm. for all the screens. A number of screens were made up in this way and in all cases the measured and the calculated wave-lengths were in close agreement.

III. The Effect of a Change in the Width of the Resonators on the Wave-Length of the Energy Reradiated by Them.

It was now possible to take up the question of the effect of the width of the resonator on the wave-length. In this investigation it was found possible to determine the limiting dimensions of what may be called a "linear resonator." It was found that a change in the width changes the wave-length but slightly. The damping, however, and the amount of energy radiated by a set of resonators depend very largely upon the width of the resonators.

The effect of the width on the wave-length was first investigated by a method differing essentially from the multiple reflection method. The method was one of tuning, in which the primary source of the energy was tuned to the resonator screens. The energy which was transmitted, *i. e.*, passed through the screen, fell upon a receiver (non-selective): the tuning was judged from the relative amounts of energy absorbed when sources of different wave-length were used. This method was however unsatisfactory and had to be abandoned, as the effect on the wave-length of the changes in width is too small to be determined in this way. The method of multiple reflection was then employed.

Resonators 24 mm. wide and 36 mm. long were placed on screens with a distance between the ends of 10 mm. and a distance of 66 mm. between adjoining sides, *i. e.*, 90 mm. between centers of the resonators measured perpendicular to the electric oscillation. An interference curve was taken with the Boltzmann mirrors, with the energy successively reflected from two of these screens. The resonators were then made narrower by cutting equal amounts of foil from the sides of each resonator. The distance between the sides was in this way changed by the amount of foil taken from a resonator, the distance between the centers remaining the same. An interference curve was taken for the altered system of resonators. The resonators were then narrowed still more and the interference

curve again taken. This process was continued until the resonators were too narrow to allow of further cutting from the sides. This set of screens will be referred to as "Set 1."

TABLE IV.

Width, Mm.	Wave-length Measured, Mm.	Wave-length Calculated, Mm.	Side-to-side Distance, Mm.
24	98	97	66
18	100	97	71
12	100	97	78
6	98.5	97	84
4	96.5	97	86
3	95	97	87
2	96	97	88

End-to-end distance.....15 mm.
Center-to-center distance.....90 mm.
Length of resonators.....36 mm.
Wave-length of source.....76 mm.
Two reflections used.

Table IV. gives the dimensions and distributions of the resonators for Set 1, and the observed wave-lengths taken from the interference curves. It will be seen from column I. that the width varied from 24 mm. to 2 mm., and from column II. that the wave-length varied from 95 mm. to 100 mm. only. The small variations in the wave-length do not follow the changes in the width. For example, with resonators having a width of 18 mm. the wave-length is 100 mm. while a width of 24 mm. gives a wave-length of 98 mm., a width of 3 mm. a wave-length of 95 mm. and a width of 2 mm. a wave-length of 96 mm. It is probable that this random apparent variation in the wave-length is due to errors of observation and does not represent a true effect. The greatest deviation of any value for the wave-length from the mean value is about 2.5 per cent., the average value of the wave-length being 98 mm. The greatest deviation between any two measured wave-lengths is only about 5 per cent. for a variation in the width of some 1,200 per cent. This small and random variation in the measured wave-lengths suggested to the authors that there was no very great effect on the wave-length caused by the change in width and further that the distance between the centers of the resonators, rather than the distance between the sides, was the important factor in determining the effect of separation perpendicular to the electric oscillation. This conclusion seems to be borne out by the results obtained from the screens of a second set, called Set 2.

The screens of Set 2 were made with resonators of the same width as

those used in Set 1. With these screens the distance between the sides of the resonators was kept constant, necessitating that the distance between the centers of the resonators be varied. It was necessary with these screens to shift the resonators toward the center row each time the width was changed, by an amount equal to the change in the width of the resonators. The distance between the sides of the resonators was constant and equal to 54 mm. Table V. gives the results for the screens of Set 2.

TABLE V.

I.	II.		III.	IV.	V.
	Wave-length				
Width, Mm.	*A* Measured, Mm.	*B* Calculated,[1] Mm.	Center-to-center, Mm.	Wave-length of Source, Mm.	End-to-end, Mm.
0.3	84.6	83.9	54.3	76	10
2.0	84.6	84.6	56	76	10
3.0	83.5	85.0	57	76	10
5.0	85.0	85.0	59	76	10
9.0	86.5	87.5	63	76	10
18.0	91.0	91.0	72	76	10

[1] Calculated for "linear resonators" with same distribution as center-to-center distance of the wide ones.

Side-to-side distance constant and equal to 54 mm.

The assumption that the width does not greatly affect the wave-length and that the center-to-center distance is the important factor in determining the wave-length explains the results from the screens of Set 2 as well as those from Set 1. In the case of Set 1 this view requires that the wave-length be very nearly a constant, since the center-to-center distance was constant. And as was shown in Table IV. this was very nearly true, though the width was varied by several hundred per cent. Again this assumption requires for the screens of Set 2 that the wave-length correspond very nearly to the changes in the center-to-center distance between the resonators. This is also shown to be true, in Table V. (cf. column II., *A*, and III.). Other assumptions were made to explain these results and it was found that an assumption involving the width as an important factor and the distance between the sides as the important separation factor, necessitated the assumption of a relation between the wave-length, width and separation which was not so simple. On the other hand, the relation between the wave-length and separation on the assumption here made appears to be practically linear, a small correction being required only in case the width was large compared with

the length of the resonator. From these results it appears that the width has but a small effect on the wave-length of the reradiated energy, and that the important factor to be considered is the distance between the centers of the resonators rather than the distance between the sides.

If the width does not greatly affect the wave-length, then the wave-length for a screen of non-linear (width comparable to the length) resonators should correspond rather closely to that for a screen of linear resonators for the same distribution. The separation distance for the linear resonators must be taken the same as the distance between the centers of the wide resonators. Calculating the wave-length which should be given by a screen of linear resonators arranged to have a *C* to *C* separation of 90 mm., the center-to-center distance for all screens of Set 1, it is found that the curve Fig. 2, *A*, requires a wave-length of 97 mm. for these linear resonators. The average value of the measured wave-length for these screens was 98 mm. In Table V. a column headed "calculated wave-length" (col. II., *B*) gives the wave-lengths for the linear resonators having the *C* to *C* separation corresponding to the center-to-center distance for the wide resonators. It is seen that these calculated values agree very well with the wave-length measured for the screen. Although the width was made to vary from 0.3 mm. to 18 mm. and the center-to-center distance from 54 mm. to 72 mm., the maximum value of the deviation of any one measured value from the corresponding calculated value is about 3 per cent. The actual variation in the wave-length is some 16 per cent.

All the results tend to show that the wave-length; or the natural period of the resonators, is very nearly independent of the width. That there is a slight variation due to the change in the width is shown by the fact that the curve for the effect of the separation in the case of the wide resonators is steeper than that for the linear resonators (see Fig. 2). But the variation of the wave-length for the square resonators from that for the linear ones having the same separation (*C* to *C*) is small. In the case when the separation is as great as can be tested by the multiple reflection method, the variation was only about 10 per cent. The change in width for this case was some 1,200 per cent.

Damping.—A change in the width of the resonators exerts a marked influence on the damping of the oscillation. This damping is evident from (1) changes in the amount of energy reradiated, (2) changes in the number of maxima and minima that may be taken, (3) changes in the purity of the radiation, or the "smoothness" of the interference curves. Table VI. shows the variation in the damping constant and amount of energy radiated as the width is changed. The damping constant[1] given

[1] Klemenčič u. Czermak, Ann. d. Phys., 1893, Vol. 50, p. 179.

in the last column was calculated as explained in the previous paper. This constant has a value of .15 for the narrow resonators and increases with the width up to a value of .31 for the square resonators. The damping constant of the radiation from the source has a value of 1.36.

TABLE VI.

Width, Mm.	Energy, One Reflection.	Energy, Two Reflections.	Damping Constant.
0.3	308 divisions	112 divisions	0.15
2	458	243	0.19
3	518	279	0.19
5	565	357	0.19
9	616	413	0.22
18	522	355	0.29
36	291	110	0.31
Strips	110	23	——
Source	——	——	1.36

With this highly damped source and total reflecting surfaces in the place of the resonator screens, it was possible to get only two or three maxima and minima in the interference curve. Using the narrow, linear resonators it was possible to take as many as fifteen or twenty such maxima and minima. That there is an effect on the purity of the radiation resulting from a change in the width is quite evident from a glance at interference curves for resonators of different widths and the same length. The curves become more and more irregular as the width is increased and secondary maxima and minima become more evident especially when the source is not well tuned to the resonators. With the wider resonators there seems to be a greater tendency of the source to force its own period on the resonators.

The energy increased with the width of the resonators until the width became about one fourth of the length, 9 mm. for the resonators used. If the width was increased beyond this value the energy decreased and when the width was equal to the length (square resonators) the energy was about the same as for the very narrow resonators. The increase in the width seemed to act as an increase in the number of resonators in the field. At the same time this increase in the number of resonators increased the damping of the oscillations in them. It appears that those factors which increased the damping also decreased the amount of energy reradiated. When the width has become as great as one fourth of the length the losses just balance the gain from the increased foil.

A rather extensive study was made of the resonators having their width equal to their length, *i. e.*, 36 mm. long and 36 mm. wide. It was found

that if two reflections be used and the C to C separation be not greater than 2.5 times the length of the resonator, the interference curves were similar to those for the linear resonator; the damping however was greater. If the separation be greater than this the curves are very irregular and it is difficult to determine the wave-length from them. These irregularities in the curves become more pronounced as the separation is increased, and there seems to be a tendency for the resonator to oscillate with the period of the source. In order to assure good results and reliable interference curves, it is necessary that the incident energy have a wave-length of not less than 0.8 of the wave-length of the resonators, provided two reflections be used. With one reflection the interference curves are not very reliable, unless the source be very closely tuned to the period of the resonators. If the source have a wave-length of less than 0.7 of the wave-length of the resonators the radiation from one reflecting screen will probably correspond to that of the first overtone of the resonator.

It is to be expected that with resonators having as great a surface as the square ones there will be a certain amount of energy regularly reflected, as from a metal sheet. In order to obtain information on this point, a set of screens was made in such a way that this regularly reflected energy did not arrive at the receiving system. This was accomplished by rotating each resonator on the screens through an angle of about thirty degrees about a line through its axis parallel to the electric component of the oscillation. In this way the corresponding points in all the resonators were held in the same plane, so that the Huygens wave-front of the radiated energy was still a plane and the reradiated energy had the same path as before. The angle taken was more than enough to guarantee that this energy did not arrive at the receiving system. It was found that the interference curves were somewhat smoother, and that the energy was reduced by about 25 per cent.

The results from the square resonators therefore show that they are not very satisfactory as resonators. The damping is high, the energy is small and they show rather marked tendency to reflect the irregularities present in the incident energy. They also show a tendency to oscillate in their overtone when the tuning is not close.

In Table VII. is given the results of some work on resonators for which the width was greater than the length. The widths were varied from one to twenty times the length of the resonator. It will be seen that the wave-length measured for such resonators is in general very near to that for the source. It is evident that when the width is greater than the length there remains but little resonance effect, and there is no definite period for the resonators. The damping is high and the energy is small.

TABLE VII.

Width, Mm.	Length, Mm.	Wave-length of Source, Mm.	Center-to-center, Mm.	End-to-end, Mm.	Reflections.	Wave-length, Mm.
27	36	76	200	10	1	73
36	36	76	72	10	2	87
54	36	76	90	10	2	71
72	36	76	108	10	2	72
72	36	76	108	10	1	73
104	36	76	108	10	2	70
108	36	76	—	10	1	70
166	36	76	—	10	1	69
226	36	76	—	10	1	72.3
226	36	76	—	56	1	72
226	36	76	—	10	2	69
320	36	76	—	10	1	70
320	36	76	—	10	2	70
328	36	76	—	10	2	68.5
550	40	87	—	10	2	86
590	36	120	—	10	2	109
590	50	120	—	43	2	110
590	50	120	—	15	2	112
590	55	120	—	20	2	112
695	36	76	—	10	2	70.5

If a "linear resonator" means one whose wave-length does not change with a decrease in the width, then the results of this investigation show that a resonator may be considered "linear" so long as the width is less than one fourth of the length of the resonator. If the additional condition be imposed that the damping must also be independent of the width, the ratio of the width of the resonator to its length must be smaller than this value.

SUMMARY.

The object of the present investigation has been to determine the relation of the dimensions and distribution of the resonators to the resulting wave-length. The present study is an application of the "*Multiple Reflection Method*" described in the preceding paper by the same authors.

A summary of this investigation follows:

1. A change in the distance between the resonators measured along the direction of the electric oscillation has very little effect on the period of oscillation of the resonator. This applies to both linear and non-linear resonators.

2. For a screen of resonators the ratio of the wave-length of the energy radiated to the length of the resonators depends very largely upon the separation of the resonators measured perpendicularly to the electric

oscillation. The value of this ratio (K) is shown by the curves in Fig. 2 These curves may be used to determine the proper distribution of resonators to give any desired wave-length.

3. The effect on the wave-length of the distance between the resonators measured perpendicular to the electric oscillation is more simply expressed in terms of the center-to-center distance between the resonators than in terms of the side-to-side distance.

4. An increase in the width of the resonators of a given length has a very small effect on the wave-length reradiated.

5. An increase in the width of the resonators increases the damping of the oscillation. As a result the interference curves are irregular and the oscillation shows a more marked tendency than in the case of the linear resonators to correspond to that of the first overtone. For widths greater than one fourth of the length of the resonator, the energy decreases with an increase in the width.

6. If the width of the resonators be greater than 1.5 times the length, there is very little effect of resonance. The wave-length of the energy from the screen is determined almost entirely by the wave-length of the incident energy.

7. The ratio of the wave-length to the resonator length for a resonator not under the influence of other resonators has been shown to be greater than 2. The value estimated from this investigation is about 2.4.

8. It appears from the results of this investigation that any resonator having a width less than one fourth of its length may be taken as a "linear resonator." If the ratio be greater than this there is but a small change in the wave-length but a marked change in the character of the reradiated energy.

The authors take pleasure in expressing here their thanks to President E. F. Nichols, at whose suggestion these investigations were undertaken, and especially do they wish to express their appreciation for the many helpful criticisms and valuable suggestions of Dr. H. W. Webb throughout the work.

PHŒNIX PHYSICAL LABORATORY,
COLUMBIA UNIVERSITY,
NEW YORK CITY,
March, 1913.

Note. — Since the foregoing papers were sent to the publishers an article by K. F. Lindman has appeared in the Ann. d. Phys., April, 1913, p. 992. His results are, in general, in agreement with the results of the present work, although the methods are quite different.

SPECIFIC INDUCTIVE CAPACITY AND ATOMIC CHARGES.

BY FERNANDO SANFORD.

IN a paper published in 1911[1] an attempt was made to find the explanation of the Volta effect in the different specific inductive capacities of the metals, though at that time it was quite generally held, especially by chemists, that the Volta effect is due to some kind of electrolytic charging of the opposed metallic surfaces by air or other gases. Since that time I have been able to show[2] that the Volta effect is independent of the character of the opposed surfaces, and that the potential difference between two metals when separated and in metallic contact with the earth must accordingly be due to some characteristic property of the metals themselves.

In several recent papers I have undertaken to show that the atoms of different elements have characteristic charges which are proportional to the products of the atomic mass into the mobility of the atomic ion in electrolysis, and that it is these charges which determine the position of the metal in the voltaic series. It is the purpose of this paper to call attention to the apparent relation of these atomic charges to the specific inductive capacities of their respective elements.

Unfortunately, these charges have been calculated for only three elements whose specific inductive capacities are known in the solid or liquid state. These elements are chlorine, bromine and iodine. For these elements, the specific inductive capacity in the liquid or solid state varies as the square root of the atomic charge of the respective atom. This is shown in Table I., where e' represents the atomic charge and k the specific inductive capacity as given in Landolt & Boernstein's Tables.

TABLE I.

Element.	e'	k	e'/k^2
Cl.....	35.2	2	8.8
Br.....	82.8	3.1	8.6
I.....	129	4	8.06

[1] A Physical Theory of Electrification, Leland Stanford, Jr., University Publications.

[2] On the Nature of the Volta Effect, PHYS. REV., XXXV., 484, December, 1912.

Since the atomic volumes of Cl, Br and I are nearly equal, their electrical densities will be proportional to their atomic charges, hence the relations shown in Table I. hold as well for the electrical densities of the atoms as for their atomic charges.

Since the specific inductive capacity of metals has not been measured, differences in its magnitude can only be inferred from other properties which are known to vary with specific inductive capacity in non-metallic substances. Fortunately, the metals have a number of such properties in common with dielectric substances. One of these properties is cohesion.

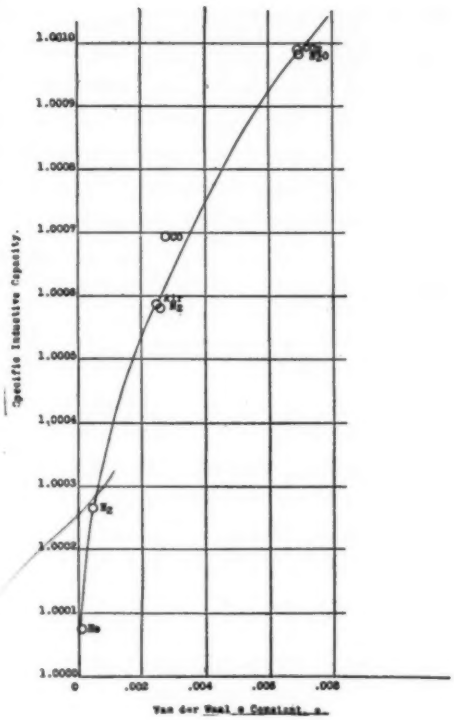


Fig. 1.

The relation of specific inductive capacity to cohesion is shown for a number of gases in Fig. 1, in which the specific inductive capacity of the gases is plotted as ordinates and the corresponding values of van der Waal's constant "a" are plotted as abscissas. The temperatures at which the specific inductive capacities were determined are not given in the tables (Kaye & Laby's) from which the data for Fig. 1 were taken. The constant "a" is calculated for the critical temperature. The data are given in Table II.

TABLE II.

Gas.	k	a
He	1.000074	.00006
H_2	1.000264	.00042
N_2	1.000581	.00259
Air	1.000586	.00257
CO	1.000695	.00275
CO_2	1.000985	.00717
N_2O	1.000990	.00710

The relation of specific inductive capacity in the solid or liquid state to cohesion may be shown with the same three elements whose specific inductive capacities are given in Table I. In a paper in Ann. d. Phys., XXXVII., 739, 1912, A. Heydweiller has calculated cohesion constants for a large number of elements. I have already shown[1] the relation of Heydweiller's cohesion constants to the characteristic charges of the atoms. Heydweiller's values for the cohesion constants of chlorine, bromine and iodine are respectively 3.02, 6.68 and 10.27. In Fig. 2 these values are plotted as abscissas and the corresponding values of the specific inductive capacities as ordinates. It will be seen from this curve that Heydweiller's values may be calculated from the specific inductive capacities by multiplying them by a constant.

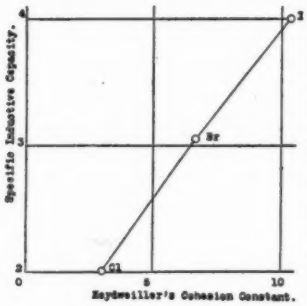


Fig. 2.

Another property of the elements which depends upon cohesion is the melting point. Both the melting point and the specific inductive capacity in the liquid or solid form are given for seven elements in Landolt and Boernstein's Tables. These are given in Table III., the melting point being given in absolute temperature.

TABLE III.

Element.	k	m.p.
Oxygen	1.465	38
Chlorine	2	171
Bromine	3+	266.7
Phosphorus	3.6	317
Iodine	4	387
Sulphur	4	387
Selenium	7.4	490

In Fig. 3 the specific inductive capacities of these elements are plotted as abscissas and their melting points as ordinates. It will be seen that with the exception of selenium the specific inductive capacities may be calculated by multiplying the melting points by a constant as closely as they may be measured by the usual methods.

The three curves already given seem to show that the cohesion between the molecules of elements vary as the specific inductive capacities of the elements. I have already shown in several papers that cohesion is an electrical force and that it varies proportionally with the characteristic

[1] On The Electrical Nature of Cohesion, PHYS. REV. I., 211, March, 1913.

charges of the molecules, hence the specific inductive capacity in the cases here considered must also vary proportionally with the molecular charges.

While the molecular charges can be calculated with a reasonable degree of probability for only a few compounds, we have in the halogen acids a group of three such compounds for which the specific inductive capacities have been determined. In a paper by Schaefer and Schlundt, Jour. Phys. Chem., XII., 669, 1909, is given a determination of the specific inductive capacities of HCl, HBr and HI in the liquid state at temperatures of from 21.7 to 27.7 degrees and also at temperatures of from − 50 to − 90 degrees. In either set of determinations the specific inductive capacity decreases at a constant rate as the melting points and the

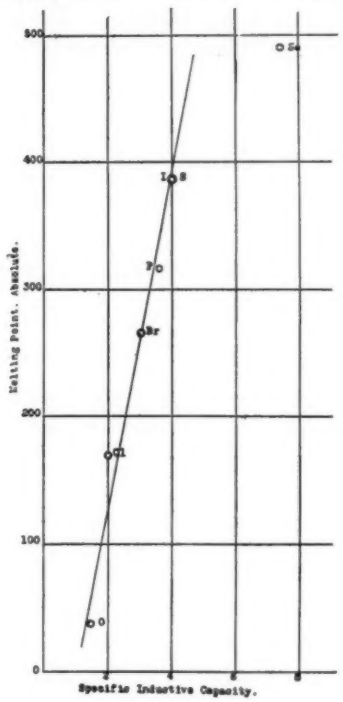


Fig. 3.

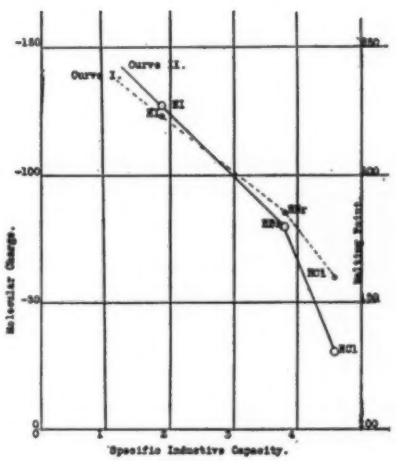


Fig. 4.

negative molecular charges (calculated as the algebraic sum of the charges of the positive and negative ion) increase. Both of these relations are shown in Fig. 4, the specific inductive capacities at the higher temperatures being used. The same relation is shown as well when the specific inductive capacities at the lower temperatures are used. In both curves given in Fig. 4 the specific inductive capacities are used as abscissas. In Curve I. the melting points and in Curve II. the molecular

charges are taken as ordinates. The data from which the curves are plotted are given in Table IV.

TABLE IV.

Molecule.	k	Mol. Chg.	Mltg. Pt. Abs.
HCl	4.6	−31.5	161
HBr	3.82	−79.9	186
HI	2.9	−127.5	223

The same relation between specific inductive capacity and melting point is shown in the compounds of the nitrogen group with hydrogen. In Jour. Phys. Chem., XVI., 253, 1912, Schlundt and Schaefer have given the specific inductive capacities of this group of compounds. Their values for specific inductive capacity at 15 degrees are given along with the melting points of the compounds in Table V.

TABLE V.

Compound.	k	Melting Point.
NH_3	15.9	63
PH_3	2.88	317
AsH_3	2.05	631
SbH_3	1.81	998

The data given in the preceding tables seem to show an unquestionable relation between specific inductive capacity and cohesion for groups of similar elements or compounds, and accordingly that in these cases a similar relation must exist between specific inductive capacity and atomic or molecular charges.

Another property which is generally understood to vary with specific inductive capacity is the refractive index or the refraction constant as calculated from the Gladstone or Lorentz formula. This refraction constant has been calculated by Gladstone for a large number of the elements, and was found to vary roughly with the square root of the atomic weight, a relation which also holds for the atomic charges. That it also, like specific inductive capacity, varies with cohesion is shown by the following considerations.

In Zeitsch. f. phys. Chem., VII., 140, 1891, J. W. Brühl gives a new determination of the atomic refraction for a considerable number of elements. For five of these elements van der Waal's constant "a" has been calculated and for six of them Kleeman[1] has calculated a con-

[1] Phil. Mag., XIX., 784, 1910.

stant "C_a" which determines the surface tension in the liquid state of the molecules to which the atoms belong. In Table VI. are given Brühl's atomic refractions, van der Waal's constant "a" and Kleeman's values of "C_a" for a number of elements.

TABLE VI.

Element.	At. Ref.	"a"	"C_a"
Hydrogen	1.103	.00042	1
Carbon	2.365	——	5.30
Nitrogen	2.76	.00165	—.—
Oxygen	2.328	.00273	5.94
Chlorine	6.014	.01063	8.40
Bromine	8.863	.01434	10.65
Iodine	13.808	——	15.49

The relations shown in the above table are represented graphically in Fig. 5. The refraction constants are plotted as abscissas. In Curve I. van der Waal's constant "a" and in curve II. Kleeman's constant "C_a" are plotted as ordinates.

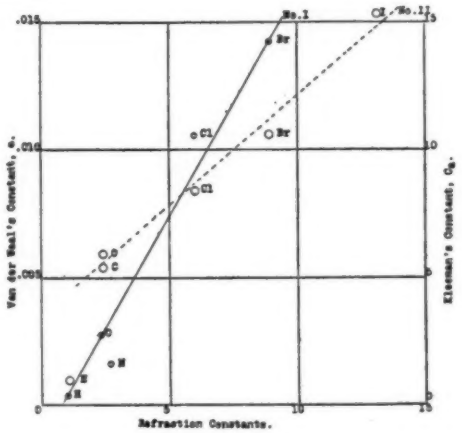


Fig. 5.

The relation of refraction constant to cohesion may be shown even in the gases of the helium group, where cohesion is weakest. In Smiles's Chemical Constitution and Physical Properties, p. 245, is given a table from Cuthbertson in which are given values of $(n - 1)10^6$ for a considerable number of elements, among them the gases of the helium group. A. O. Rankine[1] has calculated Sutherland's cohesion factor "C" for these gases from experimental data on their viscosity. The data from Cuthbertson's and Rankine's tables are given in Table VII.

[1] Phil. Mag., XXI., 45, 1911.

TABLE VII.

Element.	$(n-1)10^6$	C
He	72	70
Ne	137	56
Ar	568	142
Kr	852	188
Xe	1380	252

These relations are shown graphically in Fig. 6.

It would be easy to multiply examples showing the relation of atomic refraction to cohesion. The sole purpose in introducing this relation here is to argue from it to the relation between atomic charges and specific

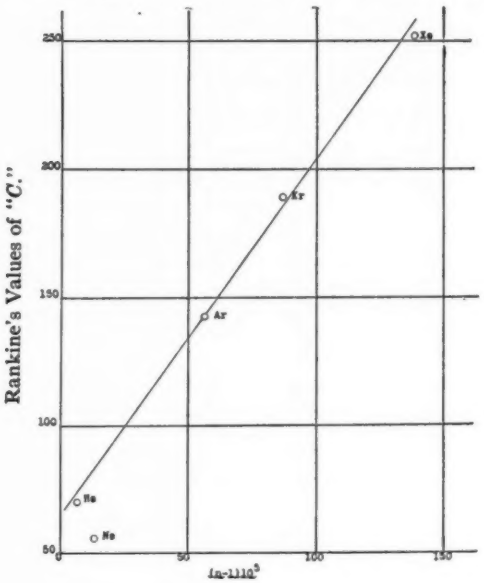


Fig. 6.

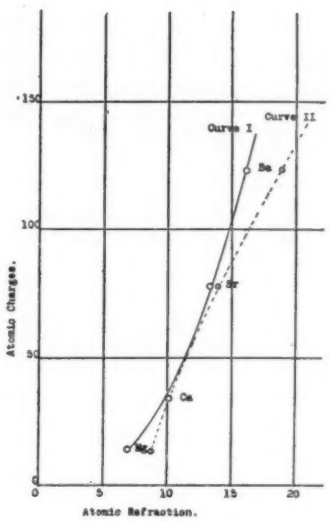


Fig. 7.

inductive capacity. I have elsewhere[1] called attention to the relation between atomic refraction and atomic charges in the alkali metals. It may not be superfluous to show here the same relation for the metals of the barium group.

Smiles has given, besides Gladstone's atomic refraction for the metals of this group a refraction equivalent for Mg, Sr and Ba determined by Pope from other considerations. Both these values as well as the atomic charges calculated from the actual velocity of the ions in electrolysis are given in Table VIII., and their relations are shown graphically in Fig. 7.

[1] Pressure-Shift of Spectral Lines, Astrophysical Journal, XXXV., 8, 1912.

Gladstone's atomic refractions are taken as abscissas in Curve I. and Pope's refraction equivalents in Curve II. The atomic charges are taken as ordinates in both cases.

TABLE VIII.

Element.	At. Ref. Pope.	At. Ref. Gladstone.	At. Chg.
Mg	8.81	6.99	13.2
Ca	——	10.1	34.8
Sr	13.95	13.3	77.5
Ba	18.94	16.1	123

The relation between atomic charges and refraction may also be shown after the atoms are combined into compound molecules. On page 272 of Ostwald's Solutions are given the refraction constants of a number of salts as calculated from the Gladstone formula $(n - 1)m/d$. These values and the sums of the atomic charges of the molecules are given in Table IX. It will be noticed that all but two of the salts are from monovalent acids, and that in the case of the two sulphates only one metallic ion has its charge counted. I do not know why this should be necessary to make the bivalent salts fall on the same curve as the monovalent salts, but I have observed that it is so.

TABLE IX.

Salt.	$\frac{n-1}{d}m$	Sum of Charges.
HCl	14.44	41.5
NaCl	15.11	52.3
KCl	18.44	76.1
HBr	20.63	90
NaBr	21.7	100.6
KBr	25.34	124.5
HI	31.17	137.5
NaI	31.59	148.2
KI	35.33	172
HNO_3	17.2	70.1
$NaNO_3$	18.7	81
KNO_3	21.8	104.7
KSO_4	30.5	143
HSO_4	22.4	108.4

The above relations are shown graphically in Fig. 8.

Another property of transparent substances which H. Becquerel has shown to be related to their refraction[1] is their magnetic rotatory power

[1] Becquerel's law is $r/n^2(n^2 - 1) = \text{const.}$ where r is magnetic rotatory power and n is the refractive index.

for polarized light. This property should accordingly be related to the specific inductive capacity.

On page 500 of Smiles's Chemical Constitution and Physical Properties is given a table of the atomic rotatory power of five metals as calculated from solutions of their salts. The calculations are by Schönrock. His

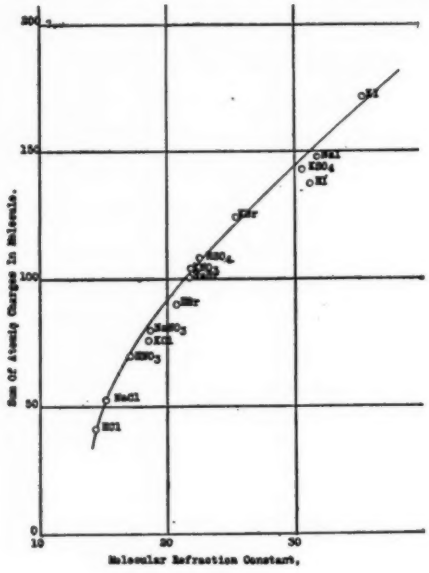


Fig. 8.

values, together with Gladstone's atomic refraction and the atomic charges, are given in Table X. It will be seen that while the magnetic rotatory power and atomic refraction vary approximately together, the magnetic rotatory power may be calculated to within the probable error

TABLE X.

Metal.	Mag. Rot.	At. Ref.	At. Chg.
Li	.398	3.6	3.6
Na	.558	4.66	15.7
Mg	.577	6.99	17.3
Ca	.691	10.1	32.5
K	.809	8	39.6

of its determination by multiplying the positive atomic charges by a constant. This relation is shown graphically in Fig. 9.

Perhaps one further relation may be presented to show that elements from the extremes of the periodic series and with opposite characteristic charges show the same relation between their charges and cohesion, viz.,

that cohesion diminishes with increasing positive charges. Landolt and Boernstein give the atomic heats of fusion of four monovalent elements whose charges may be calculated from the mobility of their ions. These elements are potassium, sodium, bromine and iodine. Their atomic heats of fusion and their characteristic charges are given in Table XI., and the relation between the two sets of properties is shown graphically

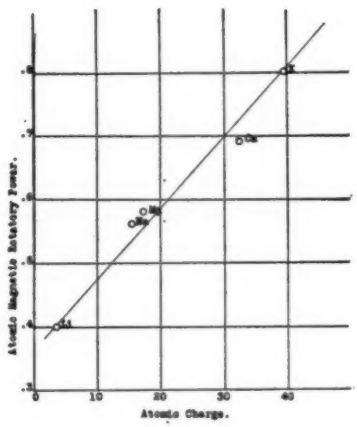


Fig. 9.

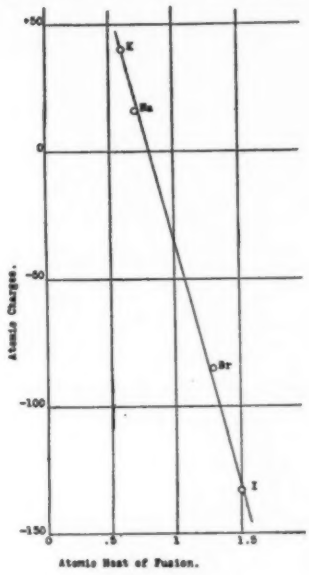


Fig. 10.

in Fig. 10. The atomic heats of fusion are taken as abscissas and the atomic charges with their appropriate signs as ordinates.

TABLE XI.

Element.	Heat of Fusion.	Atomic Charge.
K	.6	+39.6
Na	.7	+15.75
Br	1.3	−84.9
I	1.5	−132.5

SUMMARY.

In this paper and in preceding papers[1] I have undertaken to show that

[1] The following papers dealing with this question have been published: A Physical Theory of Electrification, Leland Stanford Junior Publications, May 15, 1911; On Positive Atomic Charges, PHYSICAL REVIEW, XXXII., 512, May, 1911; Atomic Charges and Cohesion, PHYSICAL REVIEW, XXXII., 518, May, 1911; Pressure-Shift of Spectral Lines, Astrophysical Journal, XXXV., 1, Jan., 1912; On Characteristic Atomic Charges and Resultant Molecular Charges, PHYSICAL REVIEW, XXXV., 276, Oct., 1912; On Convergence Frequency in Spectral Series, Astrophysical Journal, XXXVI., 255, Oct., 1912; On the Electrical Nature of Cohesion, PHYSICAL REVIEW, 2d Series, I., 211, March, 1913.

if the mobilities of ions as calculated from electrolysis be multiplied by their respective ionic masses numbers are obtained which are closely related to many of the physical and chemical properties of the ions. On the assumption that the ions in electrolysis move only short distances as free ions, and that in a water solution of given concentration the ions of different solutes are free for the same average time between dissociation and recombination, these numbers become proportional to the electric charges of the ions, and I have accordingly spoken of them as the characteristic ionic charges. Relative ionic charges which agree among themselves in the same way as those calculated from electrolysis may be calculated from the ratio e/m determined from positive ions given off by hot bodies.

These ionic charges have been shown to be related to all the properties of bodies which depend upon cohesion or affinity. They have been, for considerable groups of elements, related by definite laws to cohesion constants however calculated, to melting points, boiling points, critical points, expansion coefficients, solubilities, specific gravity, hardness, compressibility, heats of formation and heats of fusion, both in elementary substances and compounds. They have also been shown to be closely related within a given group to the pressure-shift of spectral lines, and the convergence frequencies of spectral series have been calculated from them.

In the present paper their relation to specific inductive capacity, refractive power and magnetic rotatory power have been shown in some cases, while a number of other properties which have previously been related to them have been shown to be related also to specific inductive capacity and to refractive power.

There can be no doubt that these atomic charges are actual physical quantities which are widely related to the physical and chemical properties of the atoms. Their relation to specific inductive capacity in the non-metallic elements is such that a high positive charge is associated with a high specific inductive capacity and *vice versa*, and the indications are very strong that a similar law holds in the case of the metals. This would seem to indicate that the electric elasticity of the ether is greater around negative electrons than around or within positive sub-atoms of the elements, and that in metals as in non-metals it is greater around some kinds of positive atoms than around others.

STANFORD UNIVERSITY,
February, 1913.

A STUDY OF THE REVERSIBLE PENDULUM.

Part III.[1] A Critique of Captain Kater's Paper of 1818.

By John C. Shedd and James A. Birchby.

CAPTAIN KATER's problem was the determination of a standard of length. To this end he proposed to use the dimensions of some form of physical pendulum vibrating seconds in the latitude of London. In speaking of his investigations he says, "I imagined that the least objectionable mode of proceeding would be to employ a rod drawn as a wire, in which, . . . the center of oscillation, as is well known, would be very nearly at the distance of two thirds of the length of the rod from the point of suspension; and I proposed by inverting the rod, and taking a mean of the results in each position to obviate any error which might arise from a want of uniformity in density or figure." This plan, he goes on to say, proved disappointing by reason of the impossibility of obtaining a rod sufficiently uniform. Abandoning his original plan he cast about for some other property of the pendulum that might lend itself to his purpose and he soon hit upon another form of interchangeability; he says: "It is known that the centers of suspension and oscillation are reciprocal."

This property was first pointed out by Huygens in 1673 but appears to have been first used experimentally by Kater in 1817.

Having recurred to this theorem of Huygens, Kater next noticed that the distance between these points depended upon the figure of the pendulum that "if the arrangement of its particles be changed, the place of the center of oscillation will also suffer a change."

His next suggestion is to have two fixed knife-edges and a movable mass the adjustment of which is to bring the coincidence of the knife-edges with the centers of suspension and oscillation respectively. Having so done the distance between the knife-edges becomes the length of the equivalent simple pendulum and the problem is solved.

In making his pendulum Kater had three adjustable masses the larger of which was supposed to remain in position throughout the work. A near adjustment was made by means of the second mass and the final

[1] Presented at the Cleveland meeting of the Physical Society, December 30, 1912.

adjustment secured by means of the smallest one, which he calls "the slider."

Kater had evidently carefully considered the position of the various masses for he says in italics: "*Whatever alteration may be made in the ARRANGEMENT of the weights, the effect on the vibrations (except in one particular instance) will be the same in both positions of the pendulum, always increasing or diminishing their number in both cases, though in different degrees: and the vibrations will be least affected by such change when the great weight is below, and will consequently be the nearest the truth in this position.*" From this he concludes that the adjustment secured by the first movable weight must be such that "the number of vibrations . . . must be left *in defect*" for "there is a point in the pendulum where the effect of the slider (the smaller of the movable weights) in increasing the number of vibrations is a maximum: and it appears from Dr. Young's investigations that this point in one position of the pendulum is different from that in the other. *Very near* either of these points, the pendulum being in its corresponding position, the motion of the slider produces scarcely any change in the number of vibrations; but the slider being then more distant from the point of maximum belonging to the other position of the pendulum, the corresponding increase of the number of vibrations arising from such motion of the slider, will, in that position be very perceptible.

"In the present instance, the point of maximum in either position of the pendulum, is about 4/10 inch below the middle, and consequently the distance of the two points from each other is about 8/10 inch."

The above may be easily understood by referring to curve *C*, Fig. 10. By "number of vibrations" Kater means number of vibrations done in 24 hours. This number will vary inversely with t, the period of the pendulum. The statement that a movement of the slider affects both periods in the same sense is equivalent to saying that the slope dt/dx of both curves is (in general) of the same sign for a given value of x.[1] By "in different degrees" is evidently meant that the values of dt/dx are different in the two cases.

The exception noted in the parenthesis is elaborated in the next following paragraph. By point of maximum effect on the vibration rate is meant the maximum point on the curve between d and n. Since t and n vary reciprocally this would be the minimum point on the d–t curve. Now the minimum points of the two curves do not coincide (save under one condition) and it is plain that between the minimum points the slopes of the two curves will have opposite signs. Also since the mini-

[1] To avoid confusion x is here put for d.

mum point falls below the "g" line (for Kater adjustment) it is clear that if the slider be adjusted so as to give points above the "g" line then one will be outside the minimum points and hence the slopes will here have the same sign. Kater's precaution "that the number of vibrations be left in defect" amounts to saying that the period shall be in excess, *i. e.*, the point on the curve shall be above the "g" line and hence the slopes (dt/dx) be of the same sign.

The balance of the italicized statement is also easily understood. It implies that the slopes of the two curves are unequal and also that the period on the curve having the smaller slope is nearer the correct or "g" period. (always bearing in mind that points above the g-line are intended). A glance at any pair of curves in Fig. 4 shows this at once.

From the data given in Kater's paper it is possible to determine with close approximation the constants of his pendulum as he used it and to construct the working equations. The data are not in very convenient form but the following values were obtained:

Summary of Dimensions of Kater's Pendulum.—Pendulum bar, of brass, 1/8 by 1½ by 50 inches, mass 3.30 lbs. On this bar were three masses or "weights," all of brass.

First weight: Cylinder 3½ in. diam., 1½ in. thick, mass 2 lbs. 7 oz. (2.437 lbs.). This weight was placed 8.9 cm. outside of one of the knife edges (K_1).

Second weight: Rectangular in shape, mass 7½ oz. (.47 lb.). This weight was finally placed at 81.56 cm. from K_1 toward K_2.

Third weight: Mass 4 oz. (.25 lb.). This weight constituted the "slider" and was moved over a small portion of the bar midway between the knife edges, this part of the bar being graduated in divisions of 1/20 in. (.05 in.).

The mass of the whole pendulum including the knife edges but exclusive of the third weight was very approximately 9.45 lbs. The mass of the movable weight being 0.25 lb. the value of R becomes

$$R = \frac{.25}{9.45} = 0.0264.$$

The distance between knife edges is the value of L and was

$$L = 39.441'' = 100.181 \text{ cm.}$$

The other constants are b and $(\alpha^2 + R\beta^2)$. In getting these, Kater's experimental data may be used but the results are in the nature of the case approximate, some shifting being resorted to to secure results consistent with Kater's final experimental values.

First it seems fairly close to conclude that the movable weight was in final adjustment at 1 in. from the mid-way point. Using this and other data the value of b was found to be 36.5+ and the value 36.56 was settled upon. The corresponding value of $(\alpha^2 + R\beta^2)$ is 2,392± and the value used was 2,391.98. The constants thus become:

$$\begin{aligned} L &= 100.181, \\ R &= 0.0264, \\ b &= 36.56, \\ (\alpha^2 + R\beta^2) &= 2{,}391.98. \end{aligned}$$

The resulting working equations are

$$t_1 = 0.0100613 \frac{d_1^2 + 141{,}235}{d_1 + 1{,}384.85},$$

$$t_2 = 0.0100613 \frac{(100.181 - d_2)^2 + 243{,}920.3}{2{,}510.23 - d_2}.$$

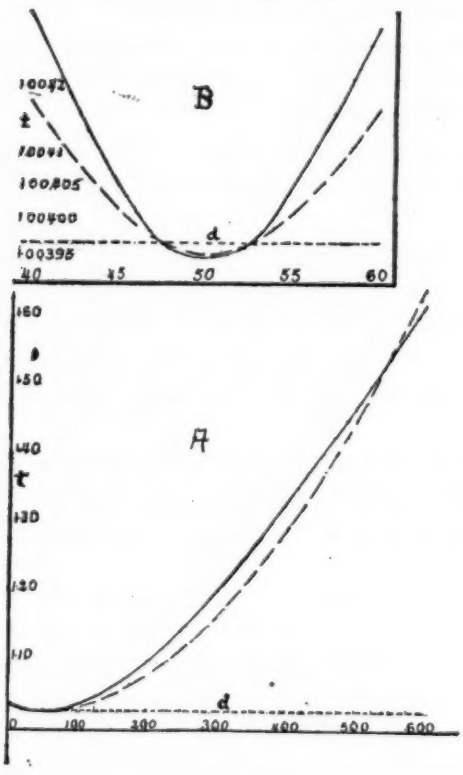


Fig. 17.

These equations are platted in Fig. 17, the lower curve showing all three intersections, while the upper curve gives the "Kater" intersections much magnified.

The following special points on the curves are of interest:
Coördinates of the three intersection points,

$$d_1 = 47.45 \text{ cm.}, \quad t_1 = 1.00396 \text{ sec.},$$
$$d_2 = 52.63 \text{ cm.}, \quad t_2 = 1.00396 \text{ sec.},$$
$$d_3 = 562.5 \text{ cm.}, \quad t_3 = 1.54038 \text{ sec.}$$

Coördinates of minimum points,

$$\delta_1 = 50.0875 \text{ cm.}, \quad \tau_1 = 1.00393 \text{ sec.},$$
$$\delta_2 = 50.1136 \text{ cm.}, \quad \tau_2 = 1.00373 \text{ sec.},$$
$$\delta_1 - \delta_2 = 0.1136 \text{ cm. or } 1/22.4 \text{ inches.}$$

In a passage already quoted Kater intimates that this distance is 8/10 in. but this value is inconsistent with his other recorded data; it must therefore have referred to the pendulum before the second weight had been finally adjusted.

The angles which the curves make with the "g" line at the I. and II. intersection points are:

First curve, $\varphi_1 = 0° \; 0' \; 2''.965$ $\qquad$ $\varphi_2 = 0° \; 0' \; 2''.86$
Second curve, $\varphi_1' = 0° \; 0' \; 2''.361$ $\qquad$ $\varphi_2' = 0° \; 0' \; 2''.26$

The angle between the

curves, *i. e.*, $\varphi_1 - \varphi_1' = 0° \; 0' \; 0''.604$, $\quad \varphi_2 - \varphi_2' = 0° \; 0' \; 0''.60$.

It will at once be seen that these curves are practically tangent and that Kater was working under conditions approximating to curve E, Fig. 10, *i. e.*, he had his pendulum very near to the limiting condition for a real intersection of the curves. As a matter of fact he was working for the first intersection (where $d < \frac{1}{2}L$) and there is nothing in his paper that directly shows whether he knew of the possibility of the adjustment of the pendulum corresponding to the second intersection to say nothing of the third. The reference to Dr. Young might imply that Young could possibly have enlightened him on both these points.

It is thus clear that Captain Kater was using the pendulum under the least favorable conditions. It also seems likely that he merely happened upon this adjustment. (This is more complimentary to him than to think that he deliberately made it by choice.) A careful examination of the problem shows that a slight change in the distribution of the masses would have more nearly given curve C, Fig. 10 and thereby have greatly improved the conditions of the experiment.

For example had the first movable weight been placed on the opposite side of the first knife edge and at the same distance from it, the improvement would have followed. The constants of the pendulum now become L and R as before. $b = 41.1$, $(\alpha^2 + R\beta^2) = 2{,}410$. The working equations are,

$$t_1 = 0.0100613\,\frac{d_1^2 + 155{,}273}{d_1 + 1{,}556.8},$$

$$t_2 = 0.0100613\,\frac{(100.181 - d_2)^2 + 223{,}591}{2{,}338.07 - d_2}.$$

The coördinates of the intersection points now become

$$d_1 = -6.45 \text{ cm.}, \quad t_1 = 1.00396 \text{ sec.},$$
$$d_2 = 106.95 \text{ cm.}, \quad t_2 = 1.00396 \text{ sec.},$$
$$d_3 = 385 \text{ cm.}, \quad t_3 = 2.092 \text{ sec.}$$

The coördinates of the minimum points are

$$\delta_1 = 29.58 \text{ cm.}, \quad \tau_1 = 0.77153 \text{ sec.},$$
$$\delta_2 = 53.78 \text{ cm.}, \quad \tau_2 = 0.9668 \text{ sec.}$$

The angles which the curves make with the "g" line at the I. and II. intersection points are

First curve,	$\varphi_1 = 0° 1' 15''.4$	$\varphi_2 = 0° 1' 10''.6$
Second curve,	$\varphi_1' = 0° 0' 54''.9$	$\varphi_2' = 0° 0' 52''.1$
	$\varphi_1 - \varphi_1' = 10''.5$	$\varphi_2 - \varphi_2' = 8''.47$

These values of $\varphi - \varphi'$ while not large are over seventeen times as large as Kater's and constitute a great improvement over his conditions of working.

Fig. 18 shows in diagram the position of the various masses as Kater

Fig. 18.

placed them (I.) and also (II.) the new position here suggested for the large weight.

To secure a still larger value for $\varphi - \varphi'$ and thereby better working conditions for using the pendulum, would require increasing the value of R; but this would be to change the pendulum beyond what could rightly have been expected of Captain Kater and the pendulum would cease to be the one with which he worked. This therefore is beyond the scope of the present paper.

In the next paper it is proposed to inquire into the best values to be given to the various constants of the various types of reversible pendulum.

December 30, 1912.

ON A METHOD OF COMPARING INDUCTANCE AND CAPACITY.

BY W. E. FORSYTHE.

IN comparing inductance and capacity by the ordinary bridge methods, it is often difficult to know just what resistances to choose. This is due to the fact that there are generally two conditions of balance, the first, the steady current balance, and the second, the one due to the reactance; the second condition, in general, depending upon the first. It often happens that with the capacity and inductance to be compared, the first balance will be so chosen that the second balance cannot be obtained. In the method outlined below this difficulty has been overcome, due to the fact that the two conditions of balance are independent.

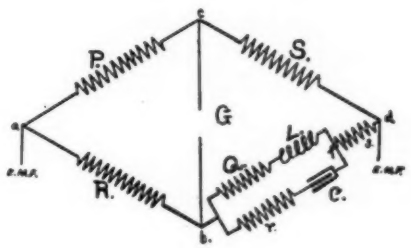


Fig. 1.

The arrangement of the circuit is given in Fig. 1, where P, R, S, Q, r, s are non-inductive resistances, L the inductance being compared with the capacity C. To obtain the conditions for balance, make P equal R, then the impedance of the arm b–d must equal that of c–d. Let Z equal the impedance of the part of the arm b–d made up of inductance and capacity, that is, b–f. We then have

$$\frac{1}{Z} = \frac{1}{Q - jX} + \frac{1}{r + jK}, \tag{1}$$

where $j = \sqrt{-1}$, $X = wL$, $K = 1/wC$, $w = 2\pi n$, n being the frequency of the alterations. As $P = R$, $(Z + s) = S$, or

$$\frac{1}{Z} = \frac{1}{S - s} \tag{2}.$$

From (1) and (2)

$$\frac{1}{Q - jX} + \frac{1}{r + jK} = \frac{1}{S - s}.$$

Or clearing

$$(S - s)(r + jK) + (S - s)(Q - jX) = (Q - jK)(r + jX). \quad (3)$$

Equating reals,

$$(S - s)r + (S - s)Q = Qr + XK. \quad (4)$$

From which

$$XK = (S - s)Q + (S - s)r - Qr. \quad (5)$$

From imaginaries

$$(S - s)K - (S - s)X = QK - rX,$$

or

$$K[(S - s) - Q] = X[(S - s) - r]. \quad (6)$$

Now it is known that for the bridge to balance, the steady current condition must hold, that is, the resistance of the arm c–d must equal that of b–d, that is,

$$S = s + Q. \quad (7)$$

From (6) and (7)

$$(S - s) = r. \quad (8)$$

From (5) and (8)

$$XK = r^2,$$

or

$$L = Cr^2.$$

Thus there are three conditions of balance, as follows:

$$S = s + Q, \quad \text{(I.)}$$
$$S = s + r, \quad \text{(II.)}$$
$$L = r^2C. \quad \text{(III.)}$$

From (I.) and (II.) it is seen that Q must equal r. Thus to balance the bridge, set $Q = r$ and then with direct current on the bridge, obtain the steady current balance, $Q + s = S$. Then with the alternating E.M.F applied to the bridge, obtain the inductive balance, with either an A.C. galvanometer or a telephone. This last balance is obtained by shifting from Q to s, keeping at the same time $r = Q$. Thus, while the bridge has three conditions of balance, the two of these due to the reactance, can be set at the same time without disturbing the steady current balance. Conditions (I.) and (II.) above might be determined directly from the bridge by noting that for the bridge to be independent of the frequency, it must hold for $w = 0$ and also for $w = \infty$. For $w = 0$ we have the condition for steady current balance, as for the steady current no current

will pass through the arm containing the condenser. For $w = \infty$ no current will pass through the inductance and the capacity will have no impedance which will make $S = s + r$.

Among the advantages of the above method, the following may be mentioned:

1. The conditions of balance are practically independent.

2. It allows a rather large range of inductance to be compared with the same capacity, without any change in the steady current balance.

3. Change of resistance to bring about the inductive balance does not materially affect the amount of current flowing through the inductance; thus the inductance can be calibrated for its entire range on the same current.

4. As the resistances in the four arms of the bridge are almost equal, the effect of the distributed capacity of the resistance boxes will be reduced to a minimum, due to the fact that the distributed capacity in each arm of the bridge will be practically the same, thus its effect will be very small. The same is true for the inductance due to the resistance coils.

The above method has been used for the past year in this laboratory and has given good satisfaction for comparing small capacities of the order of a few thousandths of a microfarad with the laboratory standard of inductance.

Department of Physics,
University of Wisconsin,
March, 1913.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE SIXTY-SEVENTH MEETING.

THE regular Spring Meeting of the Physical Society was held in Washington, D. C., on Friday afternoon, April 25, and Saturday, April 26, 1913. The Friday session was held at the Cosmos Club, President B. O. Peirce presiding. The Saturday sessions were held at the Bureau of Standards, Dr. S. W. Stratton presiding at the morning session and President B. O. Peirce at the afternoon session. At the Saturday morning session the chairman introduced to the Society Professor Willy Wien, who spoke informally on questions connected with radiation.

The following papers were presented:

The Production of Temperature Uniformity and Constancy in an Electric Furnace. A. W. GRAY.

The Determination of Thermal Expansivity at High Temperatures. A. W. GRAY.

A New Thermo Element for the Measurement of High Temperatures. C. C. BIDWELL.

Note on the Value of Throttling Experiments as a Basis for Computing Steam Tables. HARVEY N. DAVIS.

Preliminary Report on Throttling Experiments with Superheated Steam. HOWARD M. TRUEBLOOD.

Further Thermoelectric Methods in Calorimetry. WALTER P. WHITE.

A Redetermination of the Specific Heats of Platinum and Quartz. WALTER P. WHITE.

A Specific Heat Calorimeter for Small Temperature Intervals. D. R. HARPER.

A Precision Combustion Calorimeter. H. C. DICKINSON.

Note on a Calorimeter designed to Measure the Change in the Specific Heat of a Gas with Temperature. C. C. MURDOCK.

The Heat of Fusion of Ice. H. C. DICKINSON, D. R. HARPER, and N. S. OSBORN.

Improvements in the Resistance Thermometer Bridge. E. F. MUELLER.

The Comparison of Resistance Thermometers. E. F. MUELLER.

Calorimetric Cooling Corrections and the Richards "Adiabatic" Method. H. C. DICKINSON.

On the Construction of Calorimetric Thermoelements. WALTER P. WHITE.

A New Type of Temperature Regulator for Electric Furnaces. L. H. ADAMS.

Some Measurements with the Micropyrometer. G. K. BURGESS and R. C. WALTENBERG.

An Attempt to Measure in Radiation Units the Intensity of the Source Producing Photo-electric Effects for Various Wave-lengths. P. B. PERKINS.

On the Photoelectric Potentials and Currents from Thin Cathode Films. K. T. COMPTON and OTTO STUHLMANN.

The Theory of Metallic Conduction. (By title.) O. W. RICHARDSON.

Observations of Sea Water Temperatures in the Vicinity of Icebergs and in other parts of the Ocean. C. W. WAIDNER, H. C. DICKINSON, and J. J. CROWE.

On the Continued Appearance of Gases in Vacuum Tubes. GEORGE WINCHESTER and DOROTHY DUNLAP.

On Magnetic Rays. LOUIS T. MORE and S. J. MAUCHLY.

The Electric Spark. Spectrum of the Spark. WILLIAM P. SAWTELLE.

Line Spectrum from Uncharged Molecules. C. D. CHILD.

The Spectrum of Mercury in the Schuman Region. THEODORE LYMAN.

The Spectrum of Magnesium Vapor. JAMES BARNES.

The Spectrum of Active Nitrogen. (By title.) E. P. LEWIS.

A Striking Instance of the Slow Transfer of Heat through a Thin Layer of a Rarefied Gas. E. L. NICHOLS.

14. On the Maximum Value of the Magnetization Vector in Iron. B. OSGOOD PEIRCE.

A New Method of Comparing Mutual Inductances. F. B. SILSBEE.

A Vibration Electrometer. HARVEY L. CURTIS.

Ionization in Unstriated Discharge. C. D. CHILD.

Some Characteristics of Total Radiation Pyrometers. G. K. BURGESS and P. D. FOOTE.

Final Results with the Silver Voltameter. E. B. ROSA, G. W. VINAL, and A. S. MCDANIEL.

A Modification of the Carey Foster Method of Comparing Resistances. FRANK WENNER.

On the Radiation from Molten and from Solid Iron. C. C. Bidwell.

Preliminary Report of Methods for Measuring Electrolytic Resistance. F. WENNER and E. C. MCKELVEY.

Note on the Construction of Thermopiles. W. W. COBLENTZ.

ERNEST MERRITT, *Acting Secretary.*

A MODIFICATION OF THE CAREY FOSTER METHOD OF COMPARING RESISTANCES.[1]

BY FRANK WENNER.

THE method was developed for use in the measurement of the resistances of a number of similar conductors connected in series, such for example

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

as the coils of a low resistance potentiometer. In such measurements it is not possible to put a slide wire between the resistance being measured and the standard, so it is necessary to provide some other means of making and reading the small changes necessary to balance the bridge. This is accomplished by using a variable ratio coil set of the direct reading type instead of fixed ratio coils such as are regularly used in Carey Foster bridges. The ratio set used has a range of 1 per cent., and while designed for other work has been found to be very satisfactory for this purpose.

The method will be understood by reference to the figure, in which X represents the conductor whose resistance is to be compared with that of the standard S, L is a connecting link interchangeable with S, x_1 and x_2 are conductors permanently connected to X, m and n, connecting leads, A and B the two sides of the ratio set, and g and g' terminals of the conductor X. In general the same characters are used to represent the value of the resistances. We shall also let the ratio of A to B be represented by $1 + r + c$, where r is the reading and c the correction to the reading of the ratio set.

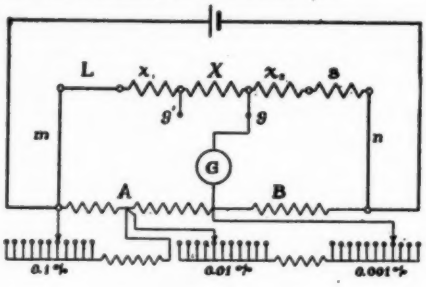


Fig. 1.

The bridge is balanced first with the arrangement shown in the figure then with the link and standard interchanged and the galvanometer connected at g'. From the two relations it follows that

$$2(X - S + L) = (r + c)(x_2 + S + n) - (r' + c)(X + x_2 + L + n) \quad (1)$$

where r' is the second reading of the ratio set. If the correction c is the same for both readings and $r + c$, $r' + c$, and L are all small in comparison with X and S then $x_2 + S + n$ is very nearly equal to $X + x_2 + L + n$ and so approximately

$$X = S - L + \frac{x_2 + S + n}{2}(r - r'). \quad (2)$$

In the case where a number of similar conductors are being compared x_1 and x_2 are of the same nominal value as X and S while m and n are small, in comparison, so equation (2) may be put in the form

$$X = N(1 + s - l + r - r') \quad (3)$$

where N is the nominal value of the resistances X and S, s the excess in proportional parts of the resistance of the standard above its nominal value, l the ratio of the resistance of the link to the resistance of the standard, and r and r' the two readings of the ratio set.

A SPECIFIC HEAT CALORIMETER FOR SMALL TEMPERATURE INTERVALS.[1]

BY D. R. HARPER, 3D.

THE calorimeter consists of a copper cup with thick walls into which are built an electric heating coil and a platinum resistance thermometer. A given quantity of electrical energy supplied to the cup and its contents is accurately measured by the aid of a potentiometer installation and a chronograph. The temperature change of the same system is determined with the thermometer.

Metallic specimens are machined to fit snugly into the hollow cylindrical interior of the cup; liquid specimens are poured directly into the cup and the equalization of the temperature is accelerated by immersing a suitable "cage" of radial sheet copper fins.

The calorimeter is well adapted to operation by the method due to T. W. Richards, where the outer jacket temperature is altered at the same rate as the calorimeter temperature. When so used, the time elapsing before the final equilibrium condition is attained may be quite long without serious disadvantage, so that the usefulness of the instrument is not confined to the better heat conductors.

The calorimeter is very similar to one devised and used by Nernst, although its inception was entirely independent and the instrument was constructed and in use before the publication of any description of Nernst's apparatus. Of the metals, copper is the only one upon which any work has been done to date, and the results of this are not ready for publication. Of other materials, some light and heavy coal tars, and some creosote oils have been measured. The results are of no general interest, but the work served very well indeed to test the usefulness of the calorimeter.

The operation of the first instrument is so successful that a second, embodying the same principles, is under construction for use in determining the latent heats of ammonia, carbon dioxide, and other liquids at various pressures, and the specific heat of these at a number of temperatures. For holding the liquids under the requisite pressures, the copper cup is provided with a lining of steel as thin as safety will permit, fitted with the necessary inlet tubes and valves.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

THE SPECTRUM OF ACTIVE NITROGEN.[1]

BY E. P. LEWIS.

THE writer in 1899 discovered a new form of afterglow in nitrogen produced by a condenser discharge with spark gap when the gas was freed from oxygen as far as possible. This afterglow had a characteristic spectrum containing a number of new bands. Further details were published in 1904,[2]

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

[2] Ann. der Phys., 2, 249, 1900; PHYS. REV., 18, 125, 1904; Astrophys. Jl., 12, 8, 1900; 20, 49, 1904.

and attention called to the remarkable fact that the vapors of mercury from the pump and from the aluminum, zinc and platinum electrodes participated in the afterglow. Strutt investigated the phenomenon on a larger scale, and gave to the glowing gas the name "active nitrogen," and Strutt and Fowler published further details on the spectrum.[1]

The writer was at first inclined to believe that small traces of oxygen were responsible for the effect, although larger quantities extinguished it; for in the earlier experiments the third group of bands, attributed by Deslandres to nitric oxide, were always present. Strutt and Fowler believed the afterglow to be an attribute of pure nitrogen, Comte favors the former view,[2] while König and Elöd[3] hold the same opinion as Strutt and Fowler.

In some recent experiments the writer failed to obtain the characteristic ultra-violet bands of the third group and those called β bands by Strutt and Fowler. The nitrogen had been for a long time in storage in the drying system. It was found that the addition of a small quantity of oxygen would cause these bands to reappear in the afterglow spectrum. They also appeared when the nitrogen was freshly prepared from sodium nitrite and ammonium sulphate, but disappeared after the gas had been for several days in contact with pyrogallic acid and the drying materials. It appears that nitric oxide is present in the nitrogen prepared in this manner, and that it is gradually absorbed. When this has happened, the third group and β bands disappear from the afterglow spectrum, and the only remaining features of the ultra-violet are the cyanogen bands, which appear only when no oxygen is present, and mercury lines. It is evident, therefore, that these bands, attributed by Strutt and Fowler to pure nitrogen, are caused by the presence of oxygen or nitric oxide. The same results were obtained with nitrogen prepared from air by passing over phosphorus.

It appears that the bands in the green, yellow and red, with heads at 5054, 5442, 5855 and 6323, which are a part of Deslandres' first group, are the only invariably characteristic features of the afterglow spectrum, all other bands so far described being due to impurities.

It seems, therefore, that Strutt and Fowler's nitrogen was contaminated with oxygen, and that their observations do not prove that the afterglow appears in perfectly pure nitrogen. This may be the case, for undoubtedly the afterglow increases in brightness as the amount of oxygen diminishes, but an amount of oxygen too small to be detected spectroscopically might possibly give rise to such an effect. It seems, therefore, still an open question whether or not "active" nitrogen is pure nitrogen.

UNIVERSITY OF CALIFORNIA,
March 10, 1913.

[1] Proc. Roy. Soc., 85, 219, 1911; 85, 777, 1911.
[2] Phys. Zeit., 14, 74, 1913.
[3] Phys. Zeit., 14, 165, 1913.

On the Construction of Calorimetric Thermoelements.[1]

By Walter P. White.

1. We have been able to obtain, from the Electrical Alloy Company, of Morristown, New Jersey, constantan wire homogeneous enough to make, without the labor of selecting and matching special portions, thermoelements of the highest precision ordinarily needed (*i. e.*, to 1/20,000) and it is probable that wire of this quality can be regularly obtained. With this wire it is only necessary to make a rapid examination, and reject the few irregular spots that occasionally occur.

2. The insulation of the junctions of a multiple element can be secured with the greatest ease and certainty merely by distributing them through a length of from one to four centimeters, according to their number. This method has no appreciable drawbacks. In general it requires that the bare junction shall not project far from the uninjured silk, but the projection can easily be kept below a millimeter if the soldering is done by dipping in a bath of solder, and if, before the wire is cut, the silk is cemented in place by shellac.

3. Inclosure in glass tubes is quick, easy, adaptable and excellent thermally. For small elements metal tubes are nearly as satisfactory in these respects, and the thermoelement, once finished, is almost indestructible.

The final result of these methods is that thermoelements of the highest sensitiveness and precision can be constructed in a few hours, without special skill or experience.

Geophysical Laboratory.

Further Thermoelectric Methods in Calorimetry.[1]

By Walter P. White.

FOR some years calorimetric determinations of great delicacy have been made by means of twin calorimeters, between which runs a very sensitive multiple thermoelement.

1. In this and similar two-body methods the smallness of the electrical quantity to be measured gives to the temperature measurement extraordinary precision combined with extraordinary ease. These methods, accordingly, while specially advantageous for precision in determining very small heat quantities, are for nearly all cases comparable with the best in convenience and certainty.

2. If the two calorimeters are alike, the external temperature and its changes affect both alike, and hence have but a slight effect upon the thermoelement reading which measures the difference in the temperatures of the two. This has sometimes been counted another great advantage of the method. A complete jacket, however, is much more effective in this direction.

3. On the other hand, there is a cooling correction error peculiar to the twin method. It arises whenever the two calorimeters are not exactly alike,

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

and can easily be shown to be $(Ka - Kb)(\theta b - \theta j)$ where Ka and Kb are the cooling rates of the two calorimeters, θb and θj the temperatures of the comparison calorimeter and of the jacket, respectively.

4. This error is nearly eliminated if three observation periods are run (as in the usual Pfaundler Method with one calorimeter), but it can be more easily avoided by making the temperature difference, $\theta b - \theta j$, small, say, below 0.2°. Only two observation periods are then needed.

5. The trouble of the temperature adjustment is largely or wholly avoided by either of two schemes, both of which utilize the low cooling rate of a vacuum-jacketed flask, which is used for the comparison calorimeter. In one scheme, a regulator keeps the jacket temperature constant. The flask is then constant to a very high degree of precision (easily to .0001°) without any further attention whatever. This has been called the constant comparison body method.

6. The other scheme avoids the regulator. A specially adjusted (shunted) thermoelement is used in determining the cooling correction. The two unlike calorimeters can then be operated as easily and conveniently as if they were twins, but the temperature adjustment is so much less exacting as to be almost negligible. This has been called the compensated calorimeter method.

7. A combination of the two methods can be made at will, either temporarily or permanently, involves no difficulties, and reduces precautions and corrections to a very low minimum.

8. The gain in simplicity and ease of construction by using vacuum-jacketed bottles is evident.

9. The best of these methods call for a complete inclosure by the jacket, but this is almost indispensable in any calorimetry of precision, and can be easily obtained. For instance, the calorimeter chamber may be a cylindrical pot, suspended in a commercial paper tub. The cover is a square box, filled with water, and closed save for two pipes, which project downward through the surface of the water below, and so provide for a complete circulation.

10. By a suitable and rather evident arrangement of thermoelements, any of these schemes can be operated with the two calorimeters in two separate jackets. This procedure is necessary in the case of adiabatic methods.

11. The great advantage of these methods is in the ease and precision of the temperature measurement. Otherwise, they present disadvantages, which, however, are evidently slight, and far outweighed by the advantages.

GEOPHYSICAL LABORATORY.

ON THE PHOTOELECTRIC POTENTIALS AND CURRENTS FROM THIN CATHODE FILMS.[1]

BY K. T. COMPTON AND OTTO STUHLMANN, JR.

RECENT results obtained by Dike[2] and by Dike and Brown[3] for the velocities of photoelectrons from thin platinum cathode films show

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

[2] PHYS. REV., Vol. 34, p. 459, 1912.

[3] PHYS. REV., Series 2, Vol. 1, p. 254, 1913.

values which are too large to be consistent with Einstein's or Richardson's theories of the photoelectric effect. We have repeated Dike's experiments with the purpose of investigating whether these unusually high velocities are real or only apparent velocities. Dike's apparatus as shown in Fig. 2 of his paper was duplicated with the addition of a small movable arm for testing contact differences of potential. This arm could be turned back so that it rested on the bottom of the chamber when the photoelectric measurements were in progress.

1. Films of platinum were sputtered on quartz in a separate vessel and were then introduced into the apparatus where velocities, saturation currents and contact differences of potential were measured. These films showed properties identical with those of ordinary polished platinum. The velocities, corrected for contact differences of potential, were equivalent to from 1.4 to 1.8 volts.

2. Films were sputtered on quartz in the test chamber and the above measurements were repeated. In this case the films did not come into contact with the air. The highest velocities obtained under these conditions were equivalent to 2.5 volts, which, when corrected for contact difference of potential, gave 5.5 volts. The current curves were similar to those obtained by other investigators.

3. Films sputtered on a polished platinum plate and investigated in the chamber in which they were sputtered gave maximum potentials, corrected for contact difference of potential, of 3.6 volts. The saturation current in this case increased logarithmically with the thickness, approaching a constant value much above that obtained from unsputtered platinum.

4. The above deposition of cathode films on polished platinum was repeated after a small quantity of soft wax had been placed on the cathode. The high velocities obtained by Dike were now obtained. Also the contact differences of potential ran practically parallel with the changes in the photoelectric potential. The maximum velocities thus obtained corresponded to 59 volts, with an accompanying contact difference of potential of 57 volts. The saturation current was not quite as large as in the preceding cases where no wax was present.

It is evident that the high photoelectric velocities are due to the presence of soft wax or grease in the apparatus. The fact that the apparent contact difference of potential between the emitting and receiving electrodes increases along with the photoelectric velocities shows that these high velocities are not true photoelectric velocities but are caused by the electrostatic attractions and repulsions of a charged insulating layer on the emitting electrode. Even the comparatively high velocities obtained when no wax was intentionally present in the chamber were doubtless due to small quantities of oil or grease unavoidably present in the metal apparatus or the ground glass joints.

There is therefore no evidence that thin metal films emit electrons with velocities different from those of electrons emitted from ordinary metals or

that they form exceptions to Einstein's and Richardson's formula

$$(\tfrac{1}{2}mv^2)_{max} = eV = h\nu - w_0,$$

which holds for ordinary metals.

PRINCETON UNIVERSITY,
UNIVERSITY OF PENNSYLVANIA.

AN ATTEMPT TO MEASURE IN RADIATION UNITS THE INTENSITY OF THE SOURCE PRODUCING PHOTO-ELECTRIC EFFECTS FOR VARIOUS WAVE-LENGTHS.[1]

BY P. B. PERKINS.

RECENT investigations seem to have established the independence of the intensity of the light and velocity of the photo-electrons produced. In opposition to this view is the work of Griffith[2] who, working with ultra-violet light falling on a negatively charged zinc plate, concluded that the ratio of current to light intensity is not constant, but increases with increasing intensity.

Lenard[3] has shown that the velocity of the electrons also depends on the quality of the light used as source.

Millikan[4] thinks the apparent difference in velocity of the electrons due to spark and arc sources, is due to the effect of long electro-magnetic waves sent out by the spark being superimposed on the ordinary photo-electric effect.

In any study of the relative photo-electric currents produced when different substances are used for electrode, it is necessary to keep the intensity of the source constant over comparatively long intervals of time. In spark sources this condition is especially difficult to meet. It would therefore seem of value to use some method of measuring the intensity of the source of the light and of noting any changes that take place as the investigation proceeds.

The literature of the subject reveals but few attempts to eliminate intensity changes and little or no effort to measure the intensity of the source directly. Thus far the writer has been unable to find any attempt to measure the intensity for various wave-lengths in the ultra-violet either for arc or discharge sources simultaneously with photo-electric effects.

Ladenburg,[5] in order to eliminate changes in intensity, used a null method in which the light fell through tubes sealed at 45° on electrodes of aluminum and copper sealed near the ends of a glass tube. The central conductor to which the electrons from both electrodes were projected was connected to earth. Light of different wave-lengths was obtained by means of mica screens of various thicknesses.

Linehop,[6] working on the photo-electric effects at low temperatures, used a

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

[2] Phil. Mag., 6, 14, 1907.

[3] Ann. d. Physik., 8, 1902, p. 149.

[4] Proc. Am. Phys. Soc., November 30, 1912.

[5] Ann. d. Physik., 12, 1903, p. 558.

[6] Ann. d. Physik., 21, 1906, p. 281.

control electrode to which a part of the light from the source was reflected from a quartz plate in the path of the rays. The constancy of the effects produced on this control was investigated from time to time.

In the earlier work of Richtmeyer[1] on photo-electric effects in a sodium cell, an incandescent light was used and its intensity varied by slips of paper. The relative intensities were judged by the comparative brightness of the sheets of paper; later[2] however an acetylene flame, having a constant intensity, was used for which the energy distribution for different wave-lengths had been previously determined by Nichols and Merritt.

Hughes[3] in obtaining photo-electric currents for compounds of zinc, tin, lead, potassium and sodium used a comparison electrode at the same angle with the arc source as the electrode on which the compounds were placed.

If photo-electric currents for different wave-lengths are to be measured, these must be corrected for differences in energy distribution for these wave-lengths. This presupposes a previous knowledge of the distribution of energy in these short wave-lengths for the source used, an extrapolation by the application of some law of radiation which is known to hold for the longer wave-lengths or a separate investigation of the energy distribution. If the intensity of the light source for these short wave-lengths could be measured in radiation units at the same time as the photo-electric effects produced, any changes in intensity would be apparent and the necessary correction could be made, a comparison of the intensity of arc, spark and other sources could be at once made in radiation units and the work of different experimenters using widely differing sources and intensities could be quantitatively compared.

It is hoped that with a very powerful source of ultra-violet light, a thermopile might be used for this purpose in series with a sensitive galvanometer or other means of measuring the small changes in electromotive force produced by the incident light. This could be compared with the effect due to a standard candle at a given distance from the thermopile. The current could be measured in amperes by obtaining the deflection produced by the current from a standard cell in series with a very large known resistance. Consequently a thermopile[4] and an astatic galvanometer were constructed after the design of Coblentz, of the Bureau of Standards. The thermopile contains twenty silver-bismuth junctions and has a resistance with connections to the key of 12.03 ohms. The galvanometer is surrounded by a series of iron shields and has a resistance of 6.48 ohms. The deflections are read by a telescope and illuminated scale at a distance of two meters. The thermopile is enclosed in a small brass cylinder closed by a brass screw cap at one end and slides on a brass collar fitting into the telescope end of a Hilger monochromatic illuminator with quartz prism. At the center of the collar in the brass base is a slit through which the light passes. When the thermopile is pulled up by a weight over a

[1] PHYS. REV., July, 1909, p. 71.
[2] PHYS. REV., March, 1910, p. 385.
[3] Proc. Camb. Phil. Soc., 16, 4, 1911, p. 376.
[4] Jour. Franklin Inst., Dec., 1911.

small pulley, the light passes through the slit to the electrode under investigation; when the weight is released, the thermopile falls of its own weight and the light passes through an extremely thin quartz window, inserted to prevent eddy currents, to the junctions.

While awaiting the arrival of the illuminator, a large induction coil with eight large condensers across the secondary inclosed in a metal box has been used to obtain the positive potential gained by a platinum electrode enclosed in a bell jar of about half a liter capacity due to light from different spark sources. The carbon arc and a quartz mercury arc have also been used. The inside of the bell jar is lined with tin covered with a thick coating of carbon, obtained by burning camphor, and connected to earth. The quartz window is 2 mm. thick. Amber insulation is used. The leads pass through iron tubes connected to earth. A Dolezalek elcterometer with a scale 2.3 meters distant is used enclosed with all keys in a metal-lined box, earth connected. The glass jar is first pumped out with an oil pump, finally with a Toepler mercury pump. The pressure is read on a McLeod gauge. The system also contains a phosphorus pentoxide drying tube and a bulb containing about eighty grams of cocoanut charcoal. It is hoped with such a large amount of charcoal, by first heating, then cooling with carbon dioxide snow, to obtain sufficiently low vacuum conditions.

A tilted Wilson electroscope is being calibrated for use if it is found that the capacity of the electrometer is too large for measuring the photo-electric effects for the shorter wave-lengths.

An attempt is also being made to construct a very sensitive capillary electrometer with the hope that, should local conditions seriously impair the sensitiveness of the galvanometer necessary for measuring effects due to the shorter wave-lengths, it may be used in place of the galvanometer.

SCIENCE HALL, HOWARD UNIVERSITY,
April 5, 1913.

THE SPECTRUM OF MAGNESIUM VAPOR.[1]

BY JAMES BARNES.

THE study of the charges which are produced in the spectrum of magnesium by different forms of electrical discharges and by varying the pressure and chemical nature of the gas surrounding the electrodes has been the subject for a number of important investigations. To these the author wishes to add a few further observations.

A piece of clean magnesium rod was placed in the center of a long silica tube which was closed with water-cooled caps and quartz plates. The tube was then placed in the core of an electric furnace and was also connected with a Geryk exhaust pump. As the magnesium slowly boiled away and condensed

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

on the sides of the tube protruding from the furnace the vapor was rendered luminous by the discharge from a large induction coil. The spectrum was photographed with a large Rowland concave grating and also at the same time with a Hilger prism spectroscope.

The negatives show that the radiations λ 2852 and λ 4571 are the strongest lines in the spectrum of magnesium vapor excited in this manner. The strong "spark line" λ 4481 is absent. The remarkable intensity of λ 4571 and the absence of λ 4481 is noteworthy. The three bands with heads at λ 5622, λ 5211, and λ 4845 are strongly depicted on the plates. These bands have been attributed to a magnesium and hydrogen compound and their wave-lengths were found by Fowler[1] to correspond with many lines in the sun-spot spectrum. These results together with an observation made by Adams[2] that the only magnesium line which shows any marked increase in the spot over the sun is λ 4571 suggests that the physical conditions which exist in magnesium vapor rendered luminous as described above are the nearest we have yet approached in the laboratory to the conditions which exist in sun-spots.

Bryn Mawr College,
April, 1913.

Preliminary Report on Throttling Experiments With Superheated Steam.[3]

By Howard M. Trueblood.

THIS paper describes an apparatus for the measurement of the specific heat and of the coefficient of the Joule-Thomson effect in steam.

The steam is generated in an 18 in. Stanley automobile boiler, gas heated, the boiler pressure being regulated to within about 0.2 lbs. per sq. in. by means of an electrical device which controls the flow of gas to the fire. From the boiler the steam passes through a lagged pipe to the main superheater, which is composed of six 30-in. lengths of double strength 2 in. wrought-iron pipe joined together by cast steel return bends. In each of these lengths of pipe is arranged a series of sheet copper discs which act as water separators, baffle the steam, and serve also as a means of conducting heat from the walls of the pipe into its interior. This superheater is gas heated. From it the steam passes through a short, heavily lagged pipe, containing a copper-gauze strainer, into a secondary superheater, and thence into and through the porous plug. This secondary superheater and the plug casing are immersed in a rapidly stirred oil bath, electrically heated, and thermostated. The secondary superheater consists of about 40 feet of three-eighth in. copper pipe, and is arranged in a coil surrounding the plug casing. The steam ordinarily enters it at a tempera-

[1] Phil. Trans. Roy. Soc., 209, A447, 1909.

[2] Astrophys. Journal, 30, 105, 1909.

[3] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

ture somewhat higher than is desired on the high side of the plug, and is cooled to temperature by the bath. The heat exchanging capacity of the secondary superheater is great enough so that, even with flows as large as 30 kilograms per hour, the temperature of the steam as it enters the superheater can be varied by 20° or 30° C. without affecting the reading of the high-side thermometer. The surroundings of the plug are therefore always at exactly the same temperature as the high-side steam.

The plug proper consists of a block of soapstone through which a considerable number of small holes are drilled. Through these holes is threaded a manganin heating wire for use in specific heat measurements. The plug is contained in a casing built up of a cast iron box and two pieces of seamless steel tubing. The steam, in its passage through the plug, and for some distance before entering and after leaving the plug, is lagged with soapstone to prevent conduction of heat inward from the bath.

The difference between the temperature of the steam on the high side of the plug and its temperature on the low side is directly determined by measuring the difference of resistance of two platinum thermometers. The drop of pressure experienced by the steam in passing through the plug is directly measured on a mercury manometer. The heat supplied to the steam while passing through the plug is determined by measuring the current supplied to the manganin heating wire and the drop of potential across the wire.

The steam, after leaving the plug, passes into a coil of pipe surrounded by cooling water, and is condensed and weighed.

Some results obtained at a single pressure and temperature as tests of the apparatus are presented.

JEFFERSON PHYSICAL LABORATORY,
CAMBRIDGE, MASS.

NOTE ON THE VALUE OF THROTTLING EXPERIMENTS AS A BASIS FOR COMPUTING STEAM TABLES.[1]

BY HARVEY N. DAVIS.

IT is the purpose of this note to show how large a part a satisfactory series of throttling experiment on superheated steam could play in increasing the accuracy of the steam tables used by engineers and others. Of the six sorts of fundamental data needed for computing a set of steam tables, two are already at hand in the literature. Of the remaining four, three could be supplied or improved, wholly or in part, more accurately by good throttling experiments than in any other way.

In the first place, the Joule-Thomson plug experiment, if carried out with a heating coil as suggested by Buckingham, would give a direct determination of c_p as well as of the Joule-Thomson coefficient μ; for a properly arranged

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

series of experiments at each high-side pressure and temperature would enable one to plot a straight line graph whose intercepts on the axes of the figure are proportional to μc_p and to μ respectively, so that the slope of the line is c_p itself.

Furthermore, a knowledge of μ, or of the product μc_p, over the whole superheated steam region would enable one to spread to higher pressures a single c_p curve, such as that at atmospheric pressure, which had been determined directly by other sorts of experiments. This can be done in three ways proposed originally by Dodge, by Davis, and by Grindley. These three processes are substantially equivalent. Their relative convenience depends on the nature and arrangement of the available Joule-Thomson data. This indirect attack on c_p is most valuable at high temperatures where direct measurements under pressure are difficult.

In the second place, a series of Joule-Thomson experiments, especially if conducted with a heating coil, affords by far the best way now available of determining the specific volume of superheated steam. The great advantage of this line of attack is that the efforts of the experimenter are expended on determining, not the whole quantity sought, but a relatively small correction term.

And in the third place, a series of Joule-Thomson experiments affords the best way now available of determining the total heat of saturated steam, and through this, the latent heat of evaporation and the specific volume of saturated steam. Here again the process involves evaluating, not the whole quantity in question, but an increment term that is never more than 5 per cent. of the whole.

The process just mentioned involves the previous knowledge of c_p near the saturation line, but it is not sensitive to errors in these assumed values of c_p, and curiously enough, one can, by means of Planck's equation, recompute c_p from the total heats thus obtained and come out with values that are much more accurate than the values from which the process started. This cycle affords a method of successive approximations for both total heats and specific heats that can be repeated as many times as may be necessary. It is almost the only satisfactory method of determining c_p on or very close to the saturation line.

JEFFERSON PHYSICAL LABORATORY,
CAMBRIDGE, MASS.

LIGHT FROM UNCHARGED MOLECULES.[1]

BY C. D. CHILD.

THE vapor passing from a mercury arc into a condensing chamber is luminous giving a line spectrum which is approximately the same as that of the arc. The light given by this vapor appears to come from uncharged molecules and not from ions, as is shown by the following.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

It is not possible that the ions are set in vibration in the *arc*, for the time that it takes for an ion to pass from the arc to the region above it is much too long for the ion to be in continuous vibration. Moreover the luminosity can be destroyed by an electric field. An electric field can remove ions from a region, but it cannot destroy the vibrations of the ions. If it removes the vibrating ions, it must remove them to some point and there must be a concentration of light in the region to which they are removed. In reality there is no concentration of light at any point.

Again it is not possible for the ionization to occur in the region above the arc, first because there is no known cause for ionization in this region. A more decisive reason is that if the ionization occurred above the arc, this region could not be cleared of light by an electric field, since a field cannot affect the uncharged molecules from which ions would be formed. But there is no other place for ionization to occur. If the light is not due to ionization occurring in either of the places, it cannot be due to ionization at all.

On the other hand if light comes from recombination of ions, it can be destroyed by removing the ions, but cannot be concentrated, since both kinds of ions cannot be concentrated in the same region. This is in accordance with the observations.

With an electric field somewhat stronger than that needed to destroy the luminosity the light is intensified and changed from a yellow to a green tint. This is due to the strengthening of the green line in the spectrum and the weakening of the yellow lines. With this stronger field there is also a large increase in the conductivity of the vapor, which is no doubt due to ionization in this region. The change in the relative intensity of the lines can be explained on the assumption that it is due to the formation of a greater number of those positive ions which lack two or more electrons.

NOTE ON THE CONSTRUCTION OF THERMOPILES.[1]

BY W. W. COBLENTZ.

WITHIN the past few years rapid strides have been made in the development of sensitive thermopiles. In this development experimenters have been greatly assisted by the theoretical principles or thermopile construction enunciated by Johansen,[2] who shows that the radii of the two wires of the element must be so chosen that the ratio between the heat conductivity and the electrical resistance is the same in both. Johansen's practice, however, is poor in that he used iron wire which rusts rapidly and in that the "cold," unexposed, junctions were joined directly to the metal posts in the frame supporting the junctions. This prevents the unexposed junctions from quickly attaining the same temperature as the "hot" junctions and hence

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

[2] Johansen, Ann. der Phys. (4), 33, p. 517, 1910.

tends to cause a "drift" as was found by Moll,[1] and more recently by the writer. The "cold" junctions should be covered with receivers having the same size and emissivity as the "hot" junctions and they should be free in the air so that there will be no drift with the variation in temperature which is caused by the incident radiation. In the Hilger thermopile which embodies the Johansen principles, the "cold" junctions appear to be joined to the metal posts in the ivory frame, hence are liable to cause a "drift" of the galvanometer needle as was found by Moll and others. It is, of course, easier to construct such a thermopile but it is not giving any forethought to the victim who must use such devices. With this thought in mind, the writer constructs his thermopiles of bismuth and silver[2] (or copper) with the "cold" junctions exposed freely in the air, and having vanes symmetrical with the "hot" junctions, which is an important, if not the most important, desideratum in thermopile construction.

The additional labor involved in constructing such a thermopile is compensated for by a greater ease of operation with a highly sensitive galvanometer, such as is required in energy measurements in the extreme ends of the spectrum.

Usually all the junctions are joined in series. However, it is just as easy to join them in series-parallel to reduce the resistance.

Flattening the bismuth wire, by pressing it between glass plates, increases the radiation sensitivity about 9 per cent.

A Vibration Electrometer.[3]

By Harvey L. Curtis.

In the comparison of very small capacities by alternating current of low frequency, using a bridge method, it is necessary to use as a detecting instrument one which will take very little current. Such an instrument can be constructed by using some of the well-known types of vibration galvanometers, but winding them of very fine wire so that the resistance will be high. However, a limit in this direction is soon reached. The vibration electrometer here described requires less current for its operation than any of the electromagnetic instruments, though the voltage which must be applied may be higher.

Greinacher[4] has recently adapted a Wulf electrometer for use as a vibration instrument. The present instrument, however, is based upon the quadrant electrometer. The vane corresponds to the needle of the quadrant electrometer, while the quadrants become small vertical plates. A horizontal section is shown in Fig. 1.

In the use of the instrument, a suitable direct current voltage is applied to the vane, while the alternating current is connected to the opposite pairs of quadrants. The vane is mounted on a bifilar suspension whose length and

[1] Moll, Dissertation, Utrecht, 1907.

[2] Coblentz, Bull. Bur. Standards, 9, p. 7, 1912.

[3] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

[4] Phys. Zs., 13, pp. 388 and 433, 1912.

tension can be readily adjusted. In this way the period of the vibrating parts may be adjusted to be the same as that of the applied alternating electromotive force. The mechanical and electrical systems are then in resonance and the amplitude of vibration will depend upon the damping of the mechanical system. As the air damping is a large part of the total, the sensitiveness is greatly in-

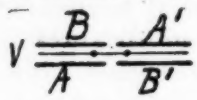


Fig. 1.

A—*A'* and *B*—*B'* are the plates to which the alternating E.M.F. is applied. *V* is the moving vane, suspended bifilarly.

creased by placing the instrument in a vacuum. In fact, the vacuum must be less than 0.1 mm. of mercury before satisfactory results can be obtained, and the sensitiveness increases in the same ratio as the pressure decreases until the pressure is as low as 0.005 mm., the lowest which was used. However, as the damping is decreased, the resonance range decreases and the response of the instrument to changes in the applied voltage becomes slower. A pressure of 0.03 mm. is as low as is desirable.

The form of the instrument is such that all of the necessary constants can be determined mathematically. This is of interest for two reasons: viz., (1) some of the assumptions in the theory of the quadrant electrometer can be demonstrated for this instrument, and (2) the instrument is a type of electrostatic motor.

The instrument has been carefully tested and experiment and theory found to agree. Some of the most important facts concerning the instruments are as follows. The natural period of the instrument depends upon the voltage applied to the vane, the period decreasing as the voltage increases. If the voltage on the vane is increased and the frequency of the alternating electromotive applied to the plates adjusted until there is a maximum deflection of the mirror, then the sensitiveness increases with increasing voltage, but when the sensitiveness becomes infinite, the period becomes zero. This shows that there is electrostatic control which is opposed to the mechanical control. The sensitiveness is inversely proportional to the damping, while the resonance range is directly proportional to the damping. The current which the instrument requires may be, under working conditions, as low as 10^{-8} or 10^{-9} amperes, while the voltage may be as low as 10^{-2} volts.

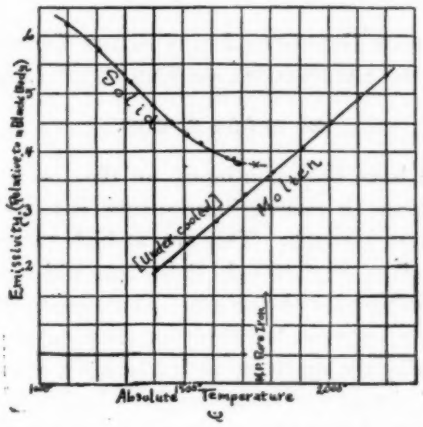
RADIATION FROM SOLID AND MOLTEN IRON.[1]

BY C. C. BIDWELL.

DEVIATIONS from full radiation for surfaces of solid and molten iron for wave-length approximately .65μ have been obtained and curves plotted showing the per cent. of true temperatures which an optical pyrometer

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 to 26, 1913.

using red light gives. Knowing the true and black-body temperatures the emissivity (relative to a black-body as unity) has been computed by means of Wien's law and the relation between the relative emissivity and the absolute temperature shown. The curve for solid iron passes over to that for molten iron at about the melting-point of pure iron. The relation between emissivity and temperature for molten iron is a linear one for the temperature range studied (1175° C. to 1900° C.). The temperature coefficient of relative emissivity for molten iron was determined as .0004243. It was found possible to undercool molten iron about 300 degrees. If we except the undercooled metal, the emissivity is a minimum at about the melting-point of pure iron.



Relation between temperature and emissivity for solid and molten iron for wave-length approximately .65 μ.

The true temperatures of the molten iron surface were obtained by sighting into a carbon cavity immersed in the iron and flush with the surface. Verification of the cavity reading as giving true temperatures was obtained for temperatures in the neighborhood of 1550° C. by means of a platinum, platinum-rhodium thermo-junction encased in a fine drawn quartz capillary and pressed into the surface of the iron. Solid iron was heated in a hydrogen atmosphere and in nitrogen. The surface of a large screw head was found convenient to sight upon the slit furnishing a good black-body. Thermo-junction tests showed the slit gave true temperatures.

CORNELL UNIVERSITY,
April 21, 1913.

A NEW THERMO-ELEMENT FOR HIGH TEMPERATURE WORK.[1]

BY C. C. BIDWELL.

A THERMAL E.M.F. between carbon and graphite has been noted and partially investigated. If the carbon and graphite is previously subjected to high heat treatment (2500° C. or more for about one minute) im-

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

purities are driven off and such samples show apparently a definite reproducible E.M.F., which varies in a definite way with the temperature. If very fine filaments are used heat conduction is reduced to a minimum. With such samples the temperature E.M.F. curve is apparently a straight line with a slope showing approximately 25.7 micro-volts per degree. A calibration has been carried to 2200° C. with no curvature apparent. There appears to be some variation with different samples of graphite and carbon. Further investigation of the junction is in progress.

CORNELL UNIVERSITY,
April 21, 1913.

ON THE OCCURRENCE OF HYDROGEN LINES IN THE TUBE-ARC AND APPLICATION TO THE NATURE OF ENHANCED LINES.[1]

BY ARTHUR S. KING.

EXPERIMENTS have been continued on the spectra given by the arc which forms when an electric-furnace tube is forced to high incandescence and made to burn through. This arc, at high current and low voltage in a partial vacuum, gives very strongly those lines which are enhanced by the spark discharge as compared with the arc. The conditions near the center of the tube's cross-section are most favorable for these enhanced lines, this being shown by photographing the spectrum with a long slit which passed across the diameter of the tube's image and observing the distribution of intensity along the line. It is now found that the hydrogen series lines, from H_α to H_ϵ inclusive, appear in this arc and are strongest at the center of the tube. The arc lines of different elements show varying degrees of response to the conditions at the center, those of titanium and vanadium being strongest close to the arcing wall.

Tests of the electrical condition of the vapor in this "tube-arc" by means of exploring electrodes have shown, first, that very vigorous ionization takes place, and second, that the ionization is not sufficient to render the vapor nearly as good a conductor as the graphite wall of the tube. These results show that the current for the most part follows the direct path between the ends of the broken tube. The greater strength of the hydrogen and enhanced lines at the center of the tube seems to be accounted for best by a strong state of electro-luminescence in this portion, caused by the ejection of high-speed corpuscles from the intensely heated carbon. If the corpuscles shot out normally from the curved wall of the tube are most effective in rendering the vapor luminous, the number of impacts by these corpuscles will be greatest near the center. The behavior of the hydrogen lines, for which the electro-luminescence of the vacuum-tube is the most favorable of the light sources usually employed, points to such an effect of electronic impacts in the tube-arc.

[1] Abstract of a paper presented at the New Haven meeting of the Physical Society, March 1, 1913.

The mixed condition of electro- and thermo-luminescence near the wall of the tube is most favorable for the lines characteristic of the arc.

A dissymmetry of the wider lines, becoming most pronounced near the center of the tube, has been a feature of the spectra throughout this investigation. The red side of the line is usually stronger, but some lines remain nearly symmetrical and λ 4481 of magnesium is unsymmetrical toward the violet. This effect, which corresponds in some degree with that shown by the condensed spark, may arise from a cause similar to that which gives displaced lines in certain stellar spectra.

MOUNT WILSON SOLAR OBSERVATORY,
February, 1913.

INDEX TO VOLUME I., SERIES II.

A.

Allen, S. J., On the Comparative Absorption of γ and X Rays, 35.

American Physical Society.
Minutes, 60, 61, 326, 466.
Treasurer's Report, 68.
Announcement of Transfer of the Review to, 1.
Abstracts, 69, 237, 326, 405, 467.

Anderson, S. Herbert, Rectifying Properties of a Photo-Electric Cell, 222.
Ionization of Potassium Vapor by Ultra-Violet Light, 233.

Arc, On the Cause of the Apparent Differences between Spark and, Sources, in the Imparting of Initial Speeds to Photo-electrons, *R. A. Millikan*, 73.

Atomic Changes, Specific Inductive Capacity, and, *Fernando Sanford*, 446.

B.

Barnes, James, The Spectrum of Magnesium Vapor, 476.

Bauer, L. A., On the Origin of the Earth's Magnetic Field, 254.

Bidwell, C. C., Radiation from Solid and Molten Iron, 482.
A New Thermo-Element for High Temperature Work, 483.

Bingham, Eugene C., Viscosity and Fluidity —A Summary of Results, II., 96.

Birchby, James A., A Study of the Reversible Pendulum. Part III. A Critique of Captain Kater's paper of 1818, 457.

Bronson Resistance, A Substitute for a, *David W. Cornelius* and *J. G. Kemp*, 80.

Brown, F. C., The Evidence that Sodium belongs to a Radio-active Series of Elements, 83.
A Method of Production of Light-Negative Selenium, 237.
The Similarity of Electrical Properties in Light-Positive Selenium to those in Certain Crystal Contacts, 245.

Brown, F. C., The Distribution of Velocities among the Photo-Electrons Emitted from Thin Cathode Films when Illuminated by a Mercury Vapor Lamp, 254.

Brownian Movements in Gases at Low Pressures, *R. A. Millikan*, 218.

Burgess, George K., A Micropyrometer, 244.

Butman, Chester A., The Electron Theory of Phosphorescence, 154.
The Actinodielectric Effect of Sulphur, 336.

C.

Calorimeter, Determination of the Heat of Solution of Metals by Means of a Vacuum Flask, *E. E. Somermeier*, 141.

Calorimeter, A Specific Heat, for Small Temperature Intervals, *D. R. Harper*, 469.

Calorimetry, Further Thermoelectric Methods in, *Walter P. White*, 471.

Capacity, On a Method of Comparing Inductance and, *W. E. Forsythe*, 463.

Carbon Contacts, Study of Resistance of, *A. L. Clark*, 50.

Carey Foster Method, A Modification of the, of Comparing Resistances, *Frank Wenner*, 467.

Carmichael, R. D., On the Theory of Relativity: Philosophical Aspects, 179.
On the Theory of Relativity: Mass, Force and Energy, 161.

Cathode Films, Note on the Aging Effect as an Influence on the number and the Velocities of Photo-Electrons from Thin, Otto Stuhlmann, Jr., 330.
On the Photoelectric Potentials and Currents from Thin, *K. T. Compton* and *Otto Stuhlmann, Jr.*, 472.

Cathode, Ray Line, On the Beaded Character of the Deflected, as Revealed by Instantaneous Photographs taken at Short Range, *Chas. T. Knipp*, 326.

Child, C. D., The Ionization in Unstriated Discharge, 408.

Child, C. D., Light from Uncharged Molecules, 479.
Clark, A. L., Study of Resistance of Carbon Contacts, 50.
Coblentz, W. W., The Constants of Spectral Radiation of a Uniformly Heated Enclosure or so called Black Body, 249.
The Constants of Spectral Radiation of a Uniformly Heated Enclosure, 334.
Note on the Construction of Thermopiles, 480.
Cohesion, On the Electrical Nature of Fernando Sanford, 211.
Cole, A. D., Diffraction and Secondary Radiation with Electric Waves of Short Wave-Length, 2.
Compton, K. T., Note on the Velocity of Electrons Liberated by Photo-electric Action, 382.
On the Photoelectric Potentials and Currents from Thin Cathode Films, 472.
Cooke, H. L., The Cooling Effect which Accompanies the Emission of Electrons from Hot Bodies, 71.
Cornelius, David W., The Velocity of Electrons in the Photo-electric Effect, as a Function of the Wave-lengths of the Light, 16.
A Substitute for a Bronson Resistance, 80,
Curtis, Harvey L., A Vibration Electrom. eter, 481.

D.

Dadourian, H. M., On a Progressive Development of the Fundamental Principles of Mechanics, 405.
Davis, Harvey N., Note on the Value of Throttling Experiments as a Basis for Computing Steam Tables, 478.
Dean, George R., A Potential and Electrostatic Force in the Field of Two Metal Spherical Electrodes. III. Charge and Capacity, 316.
Dielectrics, The Effect of, on Unipolar Induction, *E. H. Kennard*, 355.
Dike, Paul H., The Distribution among the Photo-electrons emitted from the Cathode Films when Illuminated by a Mercury Vapor Lamp, 254.
Diffraction and Secondary Radiation with Electric Waves of Short Wave-Length, *A. D. Cole*, 2.
Dimensions of Electric and Magnetic Quantities, *David L. Webster*, 329.
Discharge, The Ionization in Unstriated, *C. D. Child*, 408.
Dorsey, N. E., The Earth Inductor as an Inclinometer, 240.
Duncan, R. W. and R. C., The Optical Constants of Sodium and Potassium, 294.

E.

Earhart, Robt. F., Some Characteristic Curves for Gases at Low Pressures, 85.
Echelon Spectroscope, Line Structure and Shift with an, *Norton A. Kent*, 242.
Elasticity, The Effect of Drawing on the, of Certain Wires, *L. P. Sieg*, 70.
Electric Force, The Relation between Current and Time Rate of Change of, *Max Mason*, 83.
Electric Furnace, Summary of a Study of the, Spectrum of Iron, *Arthur S. King*, 238.
Electric Waves, Diffraction and Secondary Radiation with, of Short Wave-Length, *A. D. Cole*, 2.
Multiple Reflection of Short, from Screens of Metallic Resonators, *W. L. Severinghaus* and *W. S. Nelms*, 411.
Electrical Conductivity, Change in, due to the Orientation of Oblate Spheroids within the Conductor, *S. R. Williams*, 257.
Electrical Oscillations, The Determination of the Period of, *J. C. Hubbard*, 247.
Electrical Properties, The Similarity of, in Light Positive Selenium to those in Certain Crystal Contacts, *T. C. Brown*, 245.
Electrometer, A Method for Charging the Needle of a Quadrant, *Henry A. Erikson*, 253.
Electron Theory of Phosphorescence, The, *Chester A. Butman*, 154.
Electrons, The Cooling Effect which Accompanies the Emission of, from Hot Bodies, *H. L. Cooke* and *O. W. Richardson*, 71.
Note on the Velocity of, Liberated by Photo-electric Action, *K. T. Compton*, 382.

Electrostatic Capacities, Comparison of Small *J. C. Hubbard* and *H. F. Stimson*, 245.
Elements, Distribution of the, in the Solar Atmosphere, *Charles E. St. John*, 78.
Erikson, Henry A., A Method for Charging the Needle of a Quadrant Electrometer, 253.

F.

Falling Bodies, Air Resistance to, *A. A. Somerville*, 77.
Fluidity, Viscosity and, A Summary of Results, II., *Eugene C. Bingham*, 96.
Force, A Potential and Electrostatic, in the Field of Two Metal Spherical Electrodes, III. Charge and Capacity, *George R. Dean*, 316.
Forsythe, W. E., On a Method of Comparing Inductance and Capacity, 463.
Frye, Royal M., Vacuum Tube Discharge in a Magnetic Field, 243.

G.

Gamma Rays, On the Comparative Absorption of, *S. J. Allen* and *E. J. Lorentz*, 35.
Gases at Low Pressures, Some Characteristic Curves for, *Robt. F. Earhart*, 85.
Gilchrist, Lachlan, An Absolute Determination of the Viscosity of Air, 124.
Glowworm, The Intrinsic Brilliancy of the, *Herbert E. Ives* and *C. W. Jordan*, 69.
Graber, M. E., An Application of High Tension Oscillatory Systems to Ignition, 252.

H.

Hall Effect, The, and Some Allied Effects, *Alpheus W. Smith*, 339.
Harper, D. R., A Specific Heat Calorimeter for Small Temperature Intervals, 469.
Heat of Solution, Determination of the, of Metals by Means of a Vacuum Flask Calorimeter, *E. E. Somermeier*, 141.
Hodgman, Charles D., Notes on Problems involved in Producing Three-Color Transparencies, 258.
Hull, Albert W., A Powerful Source of Ultraviolet Light for Photo-electric Work, 329.
Hubbard, J. C., Comparison of Small Electrostatic Capacities, 245.
Hubbard, J. C., The Determination of the Period of Electrical Oscillations, 247.
Hydrogen Lines, On the Occurrence of, in the Tube-Arc and Application to the Nature of Enhanced Lines, *Arthur S. King*, 484.

I.

Inclinometer, The Earth Inductor as an, *N. E. Dorsey*, 240.
Inductance, On a Method of Comparing, and Capacity, *W. E. Forsythe*, 463.
Index of Water, Some Direct Measurements of the Temperature Correction to the Refractive, around 20° C., *F. A. Molby*, 76.
Ionization of Potassium Vapor by Ultra-Violet Light, *S. Herbert Anderson*, 233.
Ionization, The Variation of the Alpha-Ray, Of Radioactive Solids with the Thickness of the Layer, *Herbert N. McCoy*, 393.
The, in Unstriated Discharge, *C. D. Child*, 408.
Iron, Radiation from Solid and Molten *C. C. Bidwell*, 482.
Ives, Herbert B., The Intrinsic Brilliancy of the Glowworm, 69.

J.

Jones, R. J., The Annealing of Steel in an Alternating Magnetic Field, 259.
Jordan, C. W., The Intrinsic Brilliancy of the Glowworm, 69.

K.

Kemp, J. G., A Substitute for a Bronson Resistance, 80.
Kemp, J. G., Conditions of Sensibility of Photo-electric Cells with Alkali Metals and Hydrogen, 274.
Kennard, E. H., The Effect of Dielectrics on Unipolar Induction, 355.
Kent, Norton A., Line Structure and Shift with an Echelon Spectroscope, 242.
Vacuum Tube Discharge in a Magnetic Field, 243.
King, Arthur S., Summary of a Study of the Electric Furnace Spectrum of Iron, 238.
On Special Conditions in the Electric Furnace which Produce a Spectrum Similar to that of the Spark, 409.

King, Arthur S., On the Occurrence of Hydrogen Lines in the Tube-Arc and Application to the Nature of Enhanced Lines, 484.

Knipp, Chas. T., On the Beaded Character of the Deflected Cathode Ray Line as Revealed by Instantaneous Photographs taken at Short Range, 326.

L.

Langmuir, Irving, A New Vacuum Gage of Extreme Sensitiveness, 337.

Lewis, E. P., The Spectrum of Active Nitrogen, 469.

Light from Uncharged Molecules, *C. D. Child*, 479.

Lorenz, C. F., Trial of a Method for Obtaining Equality of Temperature, at High Temperatures, 332.

Lorentz, E. J., On the Comparative Absorption of γ and X Rays, 35.

M.

McCoy, Herbert N., The Variation of the Alpha-Ray Ionization of Radioactive Solids with the Thickness of the Layer, 393.

The Periods of Transformation of Uranium and Thorium, 401.

Magnesium Vapor, The Spectrum of, *James Barnes*, 476.

Magnetic Field, On the Origin of the Earth's, *L. A. Bauer*, 254.

Magnetism, The Stability of Residual, *N. H. Williams*, 360.

Magnetostrictive Effects, A Comparative Study of the Joule and Wiedemann, in Nickel Rods, *S. R. Williams*, 257.

Mason, Max, The Relation between Current and Time Rate of Change of Electric Force, 83.

Mechanics, On a Progressive Development of the Fundamental Principles of, *H. M. Dadourian*, 405.

Micropyrometer, A, *George K. Burgess*, 244.

Millikan, R. A., On the Cause of the Apparent Differences between Spark and Arc Sources in the Imparting of Initial Speeds to Photo-electrons, 73.

Brownian Movements in Gases at Low Pressures, 218.

Molby, F. A., Some Direct Measurements of the Temperature Correction to the Refractive Index of Water around 20° C., 76.

N.

Nelms, W. S., Multiple Reflection of Short Electric Waves from Screens of Metallic Resonators, 411.

A Systematic Study of Linear and Non-Linear Resonators for Short Electric Waves, 429.

Nitrogen, The Spectrum of Active, *E. P. Lewis*, 469.

O.

Optical Constants of Sodium and Potassium, *R. W.* and *R. C. Duncan*, 294.

Osborn, Frederick A., Change of Index of Refraction of Water with Change of Temperature, 198.

Oscillatory Systems, An Application of High Tension, to Ignition, *M. E. Graber*, 252.

Oxides, Resistance of, at High Temperatures, *A. A. Somerville*, 243.

P.

Pender, H., The Annealing of Steel in an Alternating Magnetic Field, 259.

Pendulum, A Study of the Reversible, Part III. A Critique of Captain Kater's paper of 1818, *John C. Shedd* and *James A. Birchby*, 457.

Perkins, P. B., An Attempt to Measure in Radiation Units the Intensity of the Source Producing Photo-electric Effects for Various Wave-lengths, 474.

Phosphorescence, The Electron Theory of, *Chester A. Butman*, 154.

Photo-electric Cell, Rectifying Properties of a, *S. Herbert Anderson*, 222.

Photo-electric Cells, Conditions of Sensibility of, with Alkali Metals and Hydrogen, *J. G. Kemp*, 274.

Photo-electric Effect, The Velocity of Electrons in the, as a Function of the Wave-Lengths of the Light, *David W. Cornelius*, 16.

Photo-electric Effects, An Attempt to Measure in Radiation Units the Intensity of the Source Producing, for Various Wave-Lengths, *P. B. Perkins*, 474.

Photoelectric Potentials, and Currents from Thin Cathode Films, *K. T. Compton* and *Otto Stuhlmann, Jr.*, 472.

Photo-electrons, On the Cause of the Apparent Differences between Spark and Arc Sources in the Imparting of Initial Speeds to, *R. A. Millikan*, 73.

The Distribution of Velocities among the, Emitted from Thin Cathode Films when Illuminated by a Mercury Vapor Lamp, *Paul H. Dike* and *F. C. Brown*, 254.

Porter, R. A., Dynamic Characteristics of the Wehnelt Rectifier, 327.

Potassium, The Optical Constants of Sodium and, *R. W.* and *R. C. Duncan*, 294.

Potassium Vapor, Ionization of, by Ultra-Violet Light, *S. Herbert Anderson*, 233.

R.

Radiation from Solid and Molten Iron, *C. C. Bidwell*, 482.

Radiation, The Constants of Spectral, of a Uniformly Heated Enclosure or so-called Black Body, *W. W. Coblentz*, 249.

The Constants of Spectral, of a Uniformly Heated Enclosure, *W. W. Coblentz*, 334.

Diffraction and Secondary, with Electric Waves of Short Wave-Length, *A. D. Cole*, 2.

Radiation Units, An Attempt to Measure in, the Intensity of the Source Producing Photo-electric Effects for Various Wave-Lengths, *P. B. Perkins*, 474.

Radiotelegraphy, Optimum Wave-Length in, *A. H. Taylor*, 321.

Rayleigh Disk, A Method of Producing Known Relative Sound Intensities and a Test of the, *G. W. Stewart* and *Harold Stiles*, 309.

Refraction of Water, Change of Index of, with Change of Temperature, *Frederick A. Osborn*, 198.

Relativity, On the Theory of, Mass, Force and Energy, *R. D. Carmichael*, 161.

On the Theory of, Philosophical Aspects, *R. D. Carmichael*, 179.

Residual Magnetism, The Stability of, *N. H. Williams*, 75.

Resistance, Air, to Falling Bodies, *A. A. Somerville*, 77.

Resonators, A Systematic Study of Linear and Non-linear, for Short Electric Waves, *W. S. Nelms* and *W. L. Severinghaus*, 429.

Richardson, O. W., The Cooling Effect which Accompanies the Emission of Electrons from Hot Bodies, 71.

S.

Sanford, Fernando, On the Electrical Nature of Cohesion, 211.

Specific Inductive Capacity and Atomic Charges, 446.

Saunders, F. S., The Series System in the Spectra of C_A, S_B, and B_A, 332.

Selenium, The Similarity of Electrical Properties in Light Positive, to those in Certain Crystal Contacts, *F. C. Brown*, 245.

A Method of Production of Light-Negative, *F. C. Brown*, 237.

Severinghaus, W. L., Multiple Reflection of Short Electric Waves from Screens of Metallic Resonators, 411.

A Systematic Study of Linear and Non-Linear Resonators for Short Electric Waves, 429.

Shedd, John C., A Study of the Reversible Pendulum. Part III. A Critique of Captain Kater's paper of 1818, 457.

Sieg, L. P., The Effect of Drawing on the Elasticity of Certain Wires, 70.

Slate, Frederick, Angle in Vector Algebra; and Composition of Rotations, 56.

Smith, Alpheus W., The Hall Effect and Some Allied Effects, 339.

Smith, N. F., The Effect of Magnetization on Thermal Conductivity, 241.

Sodium, The Evidence that, belongs to a Radioactive Series of Elements, *F. C. Brown*, 83.

The Optical Constants of, and Potassium, *R. W.* and *R. C. Duncan*, 294.

Somermeier, E. E., Determination of the Heat of Solution of Metals by Means of a Vacuum Flask Calorimeter, 141.

Somerville, A. A., Air Resistance to Falling Bodies, 77.

Resistance of Oxides at High Temperatures, 243.

Sounding Boards, the Use of, in an Auditorium, *F. R. Watson*, 241.

Sound Intensities, A Method of Producing Known Relative, and a Test of the Rayleigh Disk, *G. W. Stewart* and *Harold Stiles*, 309.

Spark, On the Cause of the Apparent Differences between, and Arc Sources in the Imparting of Initial Speeds to Photo-electrons, *R. A. Millikan*, 73.

Specific Inductive Capacity and Atomic Charges, *Fernando Sanford*, 446.

Spectra of C_A, Sr, and B_A, The Series System in the, *F. S. Saunders*, 332.

Spectrum, On Special Conditions in the Electric Furnace which Produce a, Similar to that of the Spark, *Arthur S. King*, 409.

Spheroids, Change in Electrical Conductivity due to the Orientation of Oblate, within the Conductor, *S. R. Williams*, 257.

St. John, Ancel, A Powerful Source of Ultra-violet Light for Photo-electric Work, 329.

St. John, Charles E., Distribution of the Elements in the Solar Atmosphere, 78.

Steam, Preliminary Report on Throttling Experiments with Superheated, *Howard M. Trueblood*, 477.

Steam Tables, Note on the Value of Throttling Experiments as a Basis for Computing, *Harvey N. Davis*, 478.

Steel, The Annealing of, in an Alternating Magnetic Field, *R. J. Jones*, 259.

Stewart, G. W., A Method of Producing Known Relative Sound Intensities and a Test of the Rayleigh Disk, 309.

Stiles, Harold, A Method of Producing Known Relative Sound Intensities and a Test of the Rayleigh Disk, 309.

Stimson, H. F., Comparison of Small Electrostatic Capacities, 245.

Stuhlmann, Jr., Otto, Note on the Aging Effect as an Influence on the Number and the Velocities of Photo-Electrons from Thin Cathode Films, 330.

On the Photoelectric Potentials and Currents from Thin Cathode Films, 472.

Sulphur, The Actinodielectric Effect of, *Chester A. Butman*, 336.

T.

Taylor, A. H., Optimum Wave-Length in Radiotelegraphy, 244.

Taylor, A. H., Optimum Wave-Length in Radiotelegraphy, 321.

Temperature, Trial of a Method for Obtaining Equality of, at High Temperatures, *C. F. Lorenz*, 332.

Thermal Conductivity, The Effect of Magnetization on, *N. F. Smith*, 241.

Thermoelectric Methods, [Further, [in Calorimetry, *Walter P. White*, 471.

Thermo-Element, A New, for High Temperature Work, *C. C. Bidwell*, 483.

Thermoelements, On the Construction of Calorimetric, *Walter P. White*, 471.

Thermopiles, Note on the Construction of, *W. W. Coblentz*, 480.

Thorium, The Periods of Transformation of Uranium and, *Herbert N. McCoy*, 401.

Throttling Experiments, Preliminary Report on, With Superheated Steam, *Howard M. Trueblood*, 477.

Note on the Value of, as a Basis for Computing Steam Tables, *Harvey N. Davis*, 478.

Transfer of the Review to the American Physical Society, Announcement of the, 1.

Transparencies, Notes on Problems involved in Producing Three-color, *Charles D. Hodgman*, 258.

Trueblood, Howard M., Preliminary Report on Throttling Experiments with Superheated Steam, 477.

U.

Ultra-Violet Light, Ionization of Potassium Vapor by, *S. Herbert Anderson*, 233.

A Powerful Source of, for Photo-electric Work, *Albert W. Hull* and *Ancel St. John*, 329.

Unipolar, Induction, The Effect of Dielectrics on, *E. H. Kennard*, 355.

Uranium, The Periods of Transformation of, *Herbert N. McCoy*, 401.

V.

Vacuum Gage, A New, of Extreme Sensitiveness, *Irving Langmuir*, 337.

Vacuum Tube Discharge in a Magnetic Field, *Norton A. Kent* and *Royal M. Frye*, 243.

Vector Algebra, Angle in, and Composition of Rotations, *Frederick Slate*, 56.

Ventilating Systems. The Effects of Gas Currents on Sound, with Applications to, of Auditoriums, *F. R. Watson*, 76.

Viscosity, An Absolute Determination of the, of Air, *Lachlan Gilchrist*, 124.

Viscosity and Fluidity—A Summary of Results, II., *Eugene C. Bingham*, 96.

W.

Watson, F. R., The Effects of Gas Currents on Sound, with Applications to Ventilating Systems of Auditoriums, 76.

The Use of Sounding Boards in an Auditorium, 241.

Wave-Length, Optimum, in Radiotelegraphy, *A. H. Taylor*, 244, 321.

Webster, David L., The Dimensions of Electric and Magnetic Quantities, 329.

Wehnelt Rectifier, Dynamic Characteristics of the, *R. A. Porter*, 327.

Wenner, Frank, A Modification of the Carey Foster Method of Comparing Resistances, 463.

White, Walter P., Further Thermoelectric Methods in Calorimetry, 471.

On the Construction of Calorimetric Thermoelements, 471.

Williams, N. H., The Stability of Residual Magnetism, 75.

The Stability of Residual Magnetism, 360.

Williams, S. R., A Comparative Study of the Joule and Wiedemann Magnetostrictive Effects in Nickel Rods, 257.

Change in Electrical Conductivity due to the Orientation of Oblate Spheroids within the Conductor, 257.

X.

X Rays, On the Comparative Absorption of, *S. J. Allen* and *E. J. Lorentz*, 35.

VOLUME I SECOND SERIES NUMBER 6

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Multiple Reflection of Short Electric Waves from Screens of Metallic Resonators. W. L. SEVERINGHAUS and W. S. NELMS 411

A Systematic Study of Linear and Non-Linear Resonators for Short Electric Waves. W. S. NELMS and W. L. SEVERINGHAUS 429

Specific Inductive Capacity and Atomic Charges. FERNANDO SANFORD 446

A Study of the Reversible Pendulum. Part III. A Critique of Captain Kater's Paper of 1818. JOHN C. SHEDD and JAMES A. BIRCHBY 457

On a Method of Comparing Inductance and Capacity. W. E. FORSYTHE 463

Proceedings of the American Physical Society: 466

Minutes of the Sixty-seventh Meeting; A Modification of the Carey Foster Method of Comparing Resistances, *Frank Wenner*; A Specific Heat Calorimeter for Small Temperature Intervals, *D. F. Harper, 2d.*; The Spectrum of Active Nitrogen, *E. P. Lewis*; On the Construction of Calorimetric Thermoelements, *Walter P. White*; Further Thermoelectric Methods in Calorimetry, *Walter P. White*; On the Photoelectric Potentials and Currents from Thin Cathode Films, *K. T. Compton and Otto Stuhlmann, Jr.*; An Attempt to Measure in Radiation Units the Intensity of the Source Producing Photo-Electric Effects for Various Wave-Lengths, *P. B. Perkins*; The Spectrum of Magnesium Vapor, *James Barnes*; Preliminary Report on Throttling Experiments with Superheated Steam, *Howard M. Trueblood*; Note on the Value of Throttling Experiments as a Basis for Computing Steam Tables, *Harvey N. Davis*; Light from Uncharged Molcules, *C. D. Child*; Note on the Construction of Thermopiles, *W. W. Coblentz*; A Vibration Electrometer, *Harvey L. Curtis*; Radiation from Solid and Molten Iron, *C. C. Bidwell*; A New Thermo-Element for High Temperature Work, *C. C. Bidwell*; On the Occurrence of Hydrogen Lines in the Tube-Arc and Application to the Nature of Enhanced Lines, *Arthur S. King*.

Index 486

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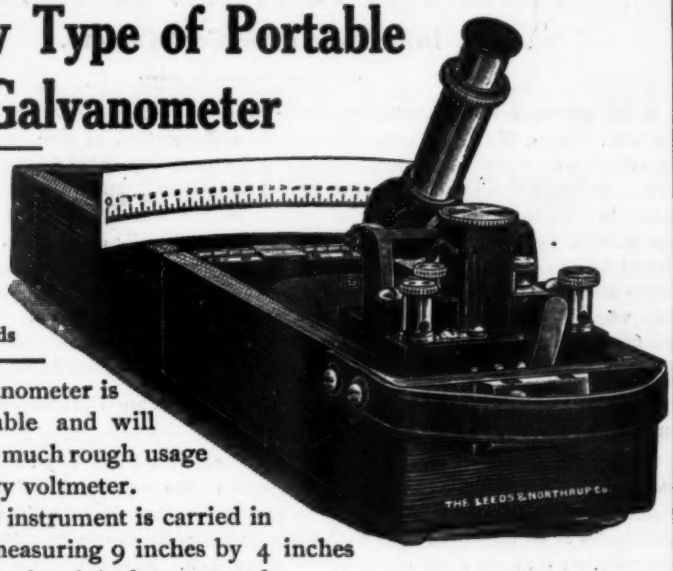

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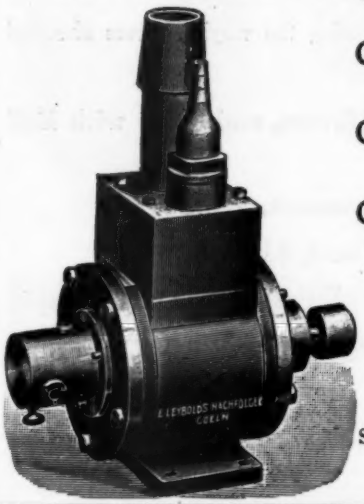

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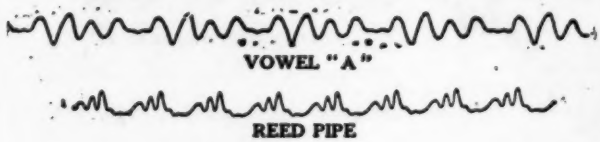

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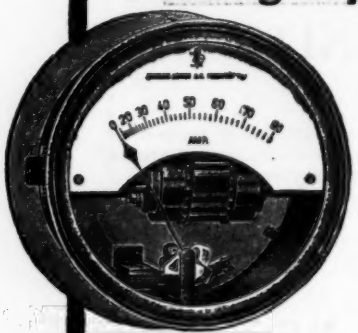

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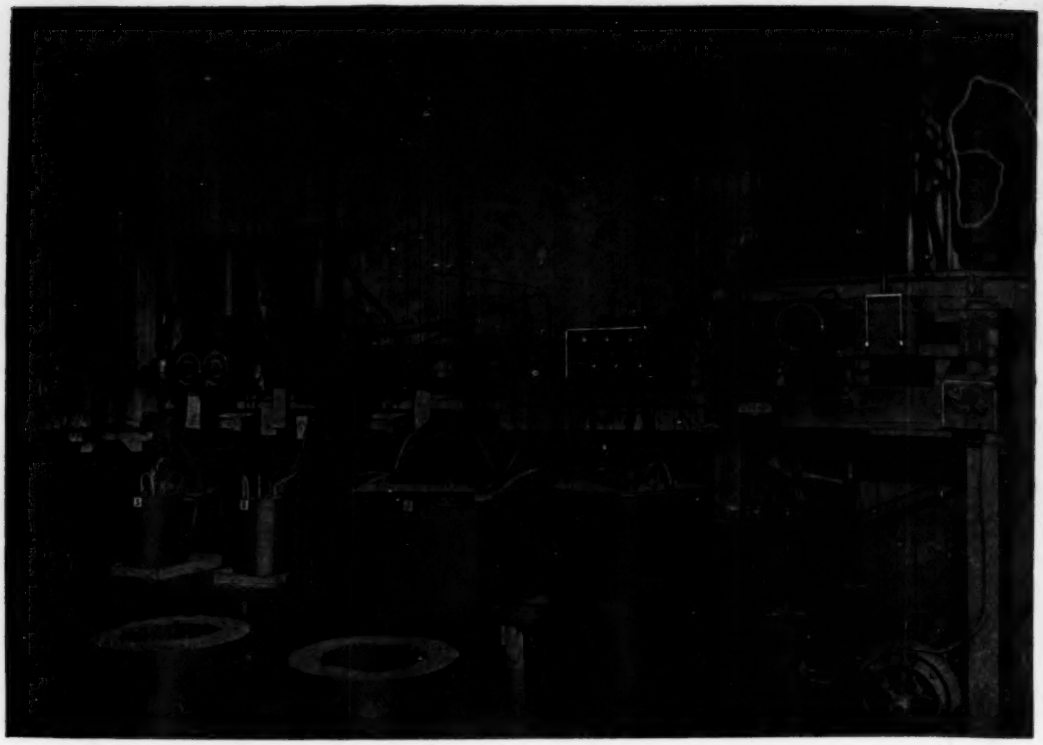

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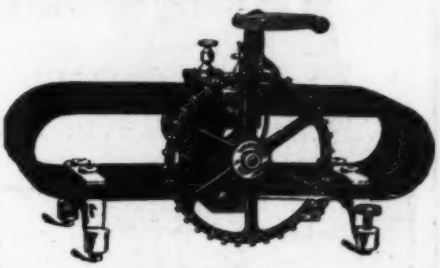

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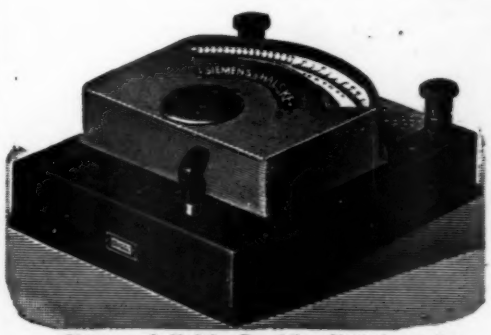

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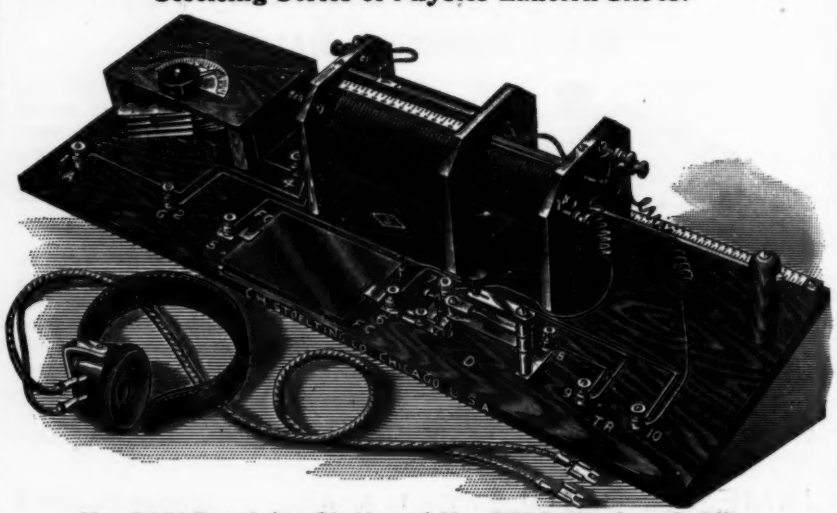

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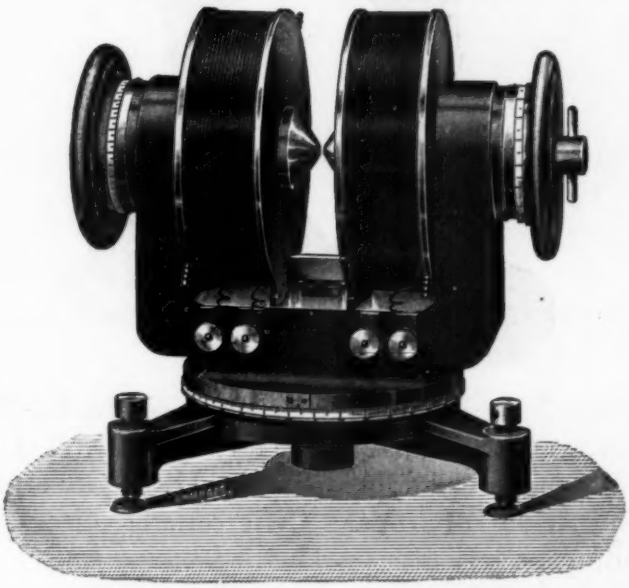

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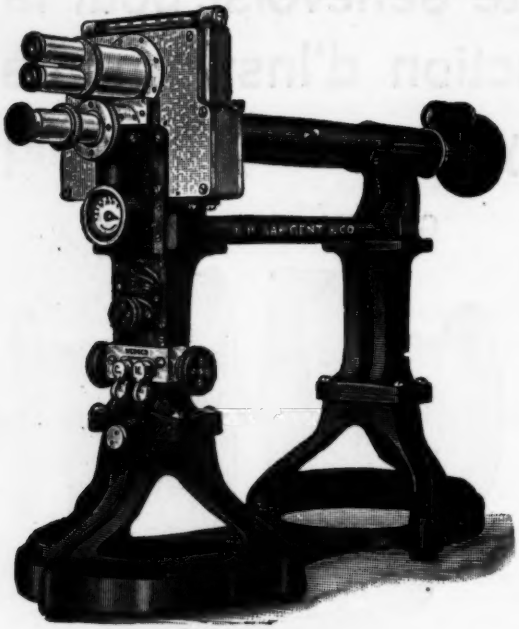

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